Supersingular Isogeny Key Encapsulation (SIKE)

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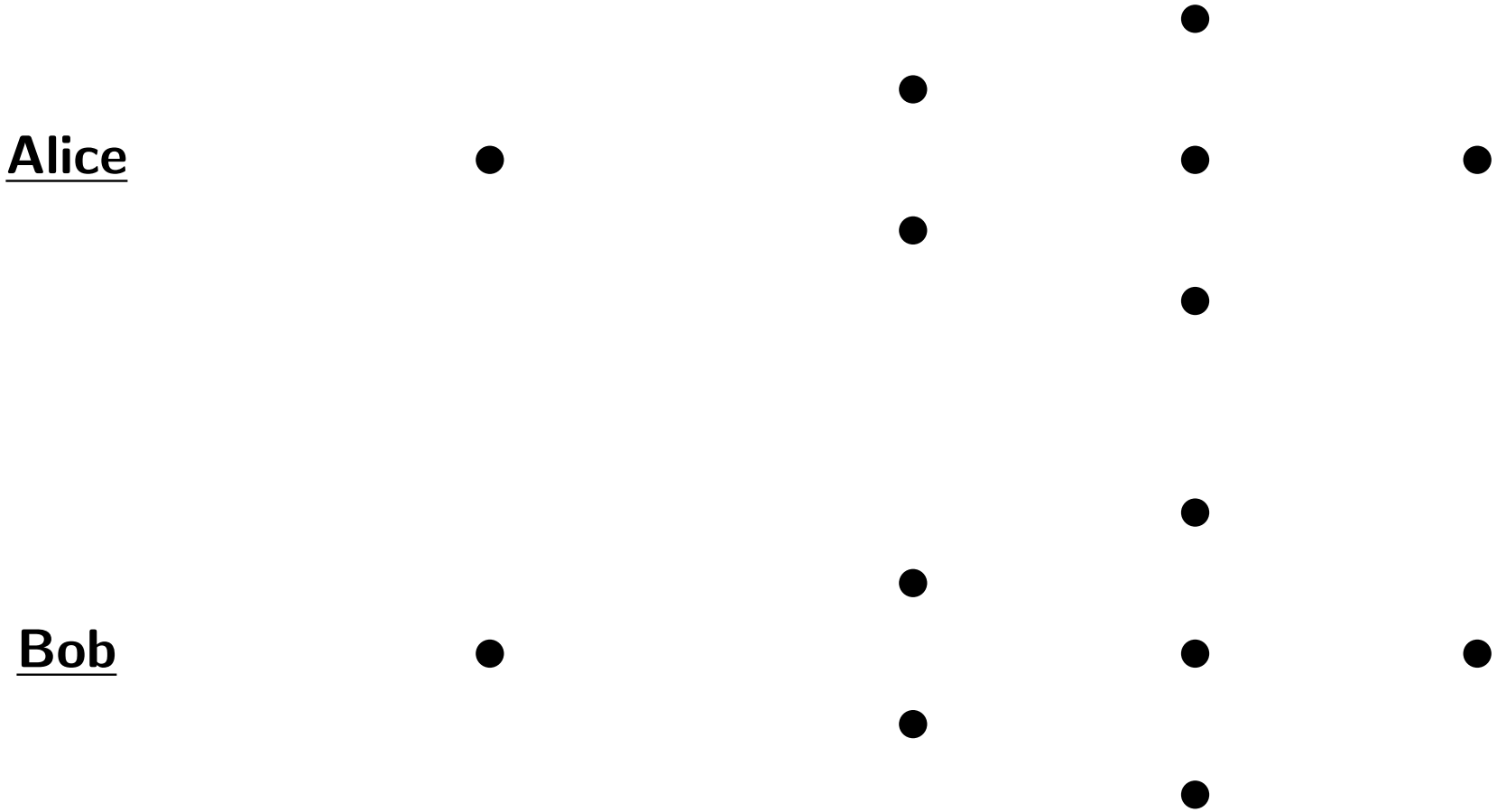
Introduction

(generic intro...)

A graph-based protocol



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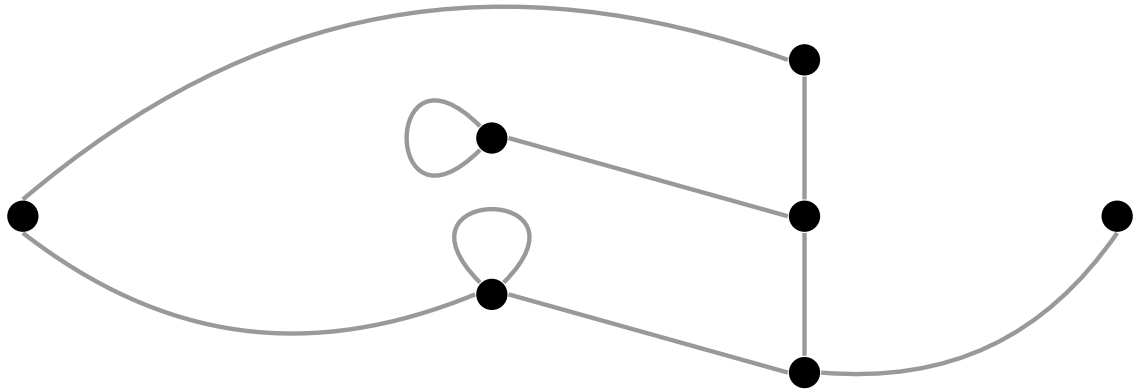

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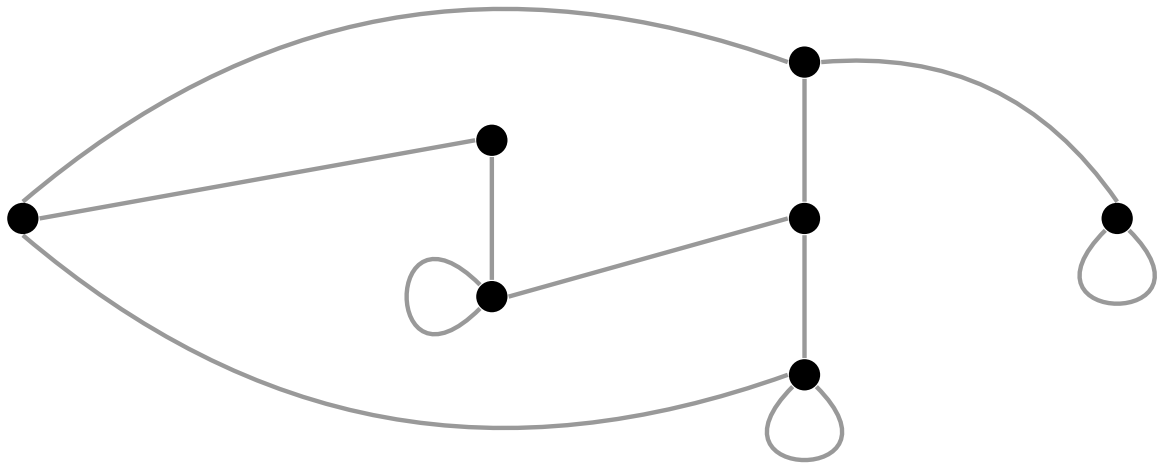

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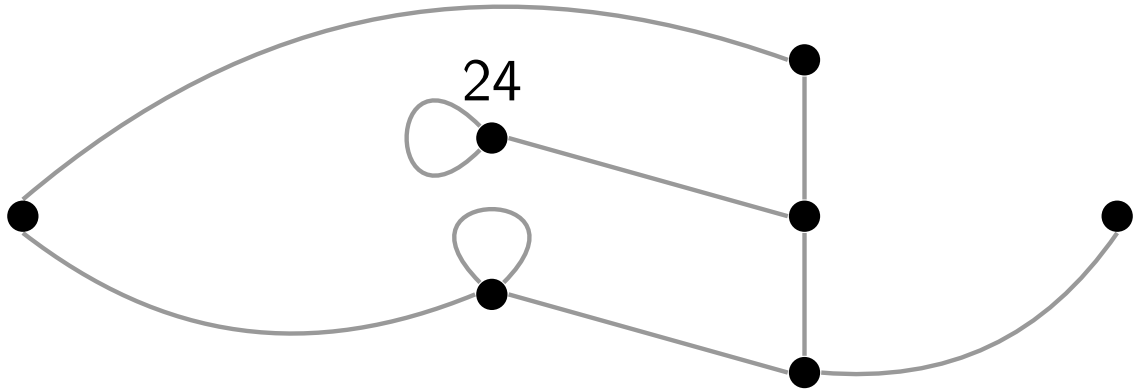

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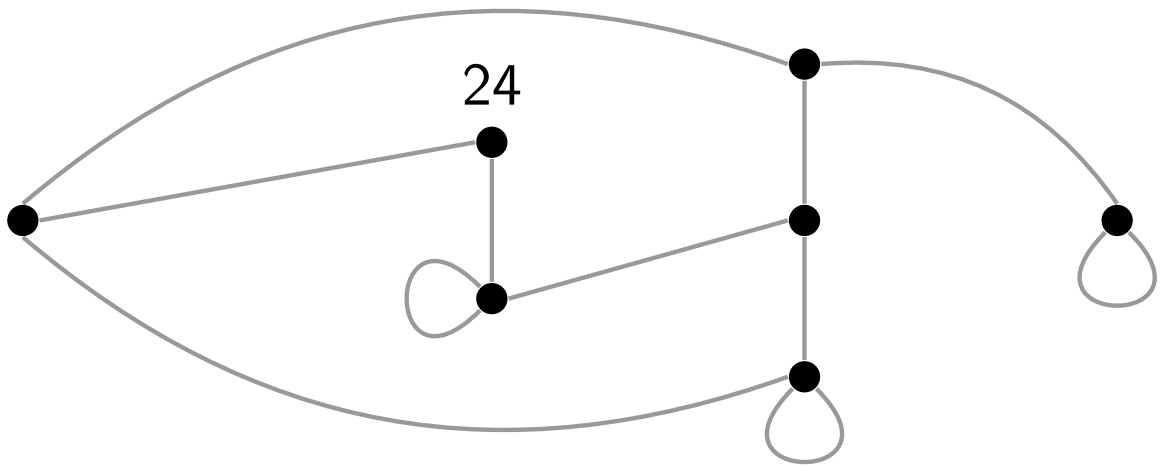

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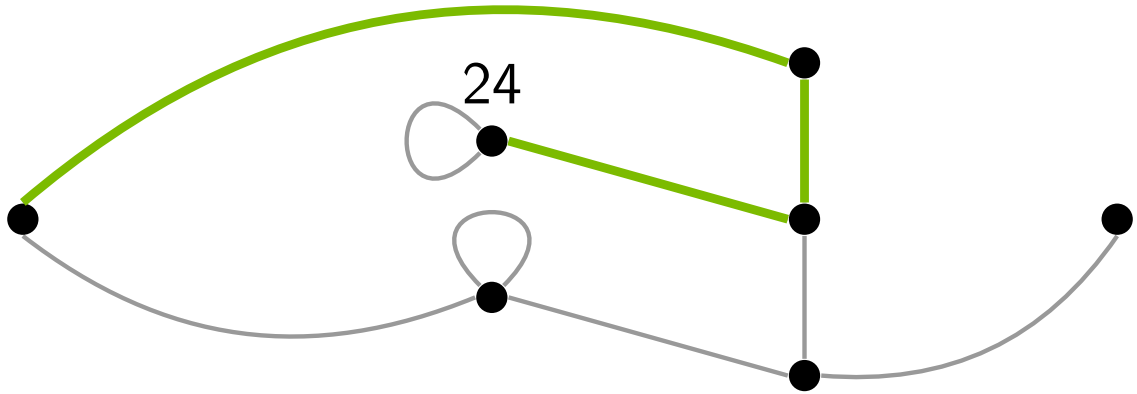

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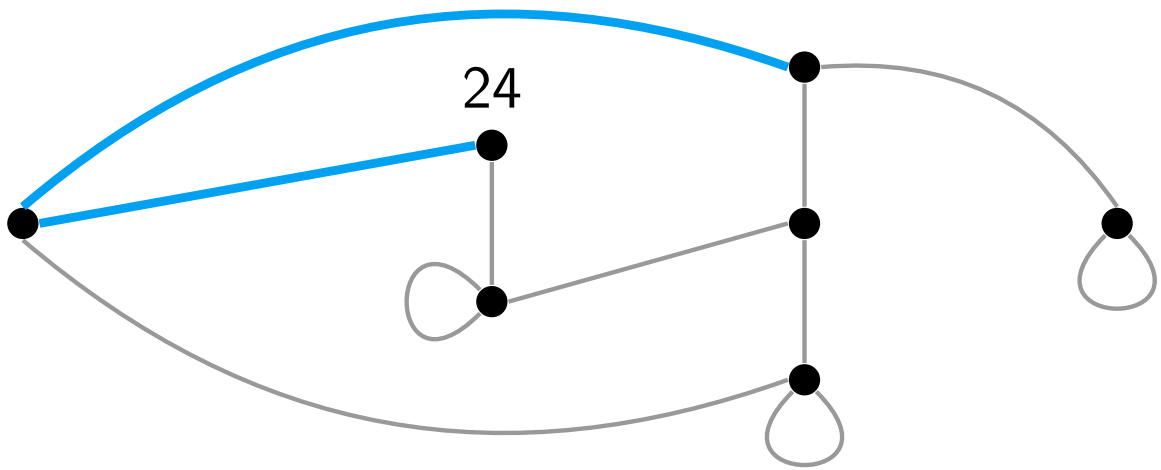

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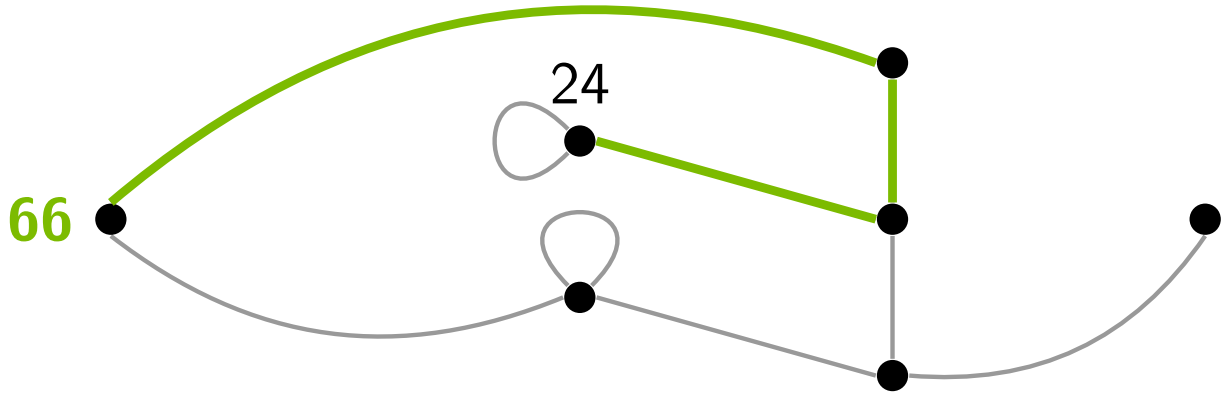

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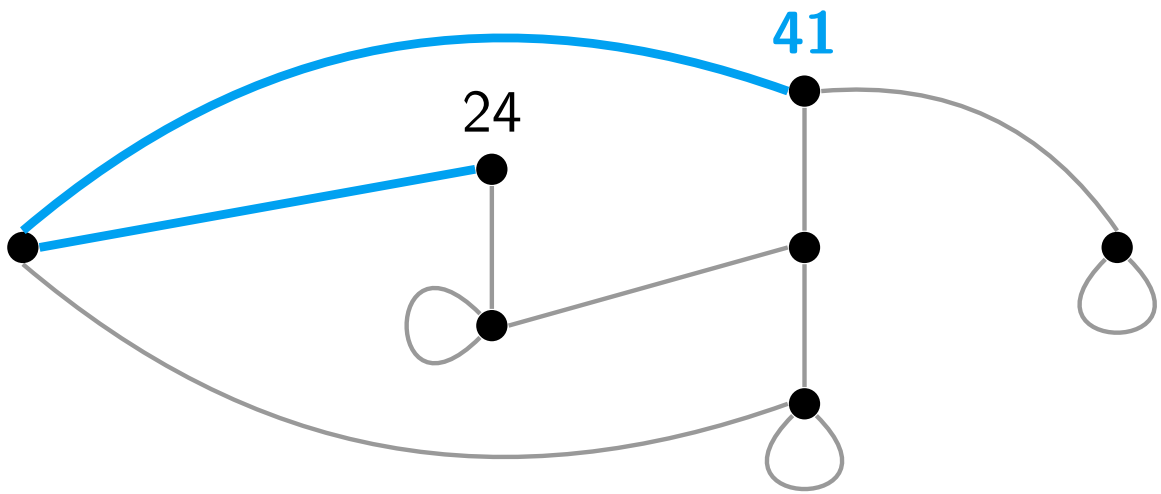

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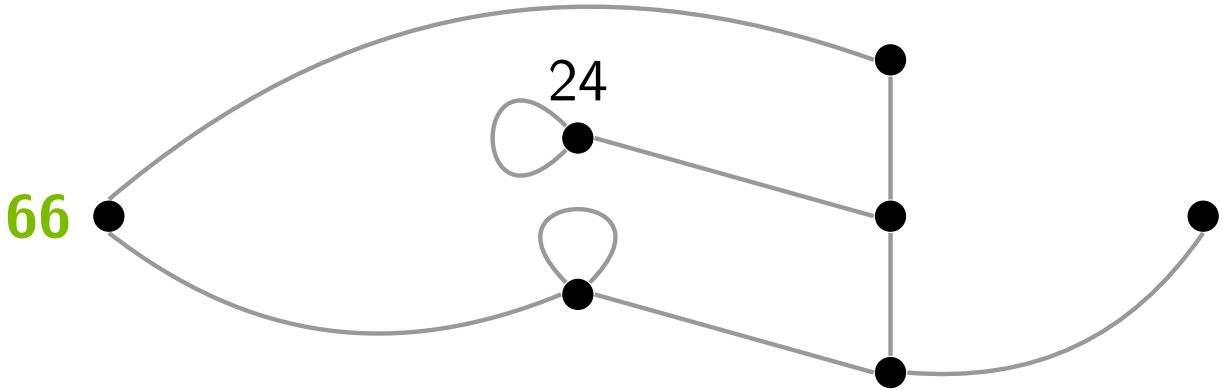

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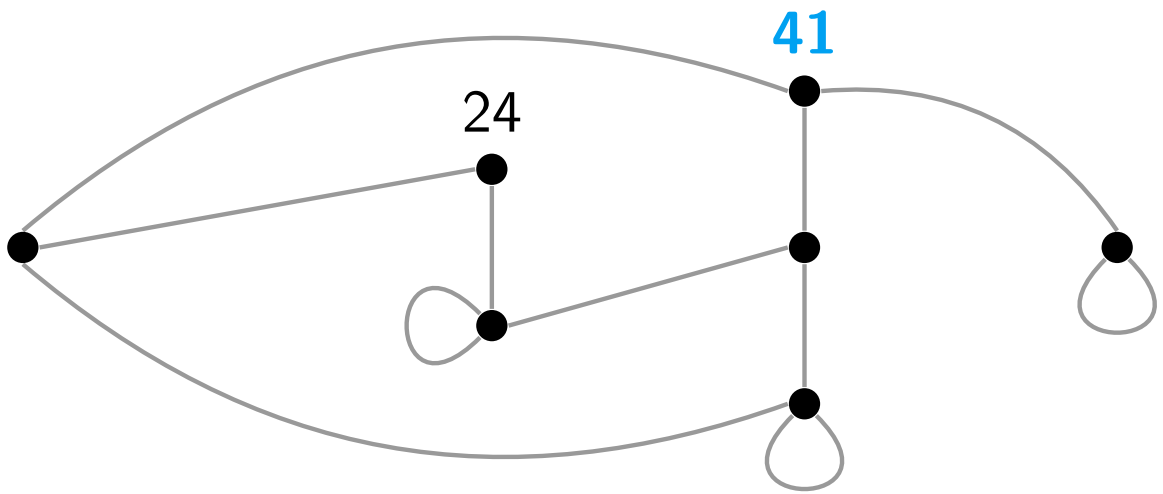

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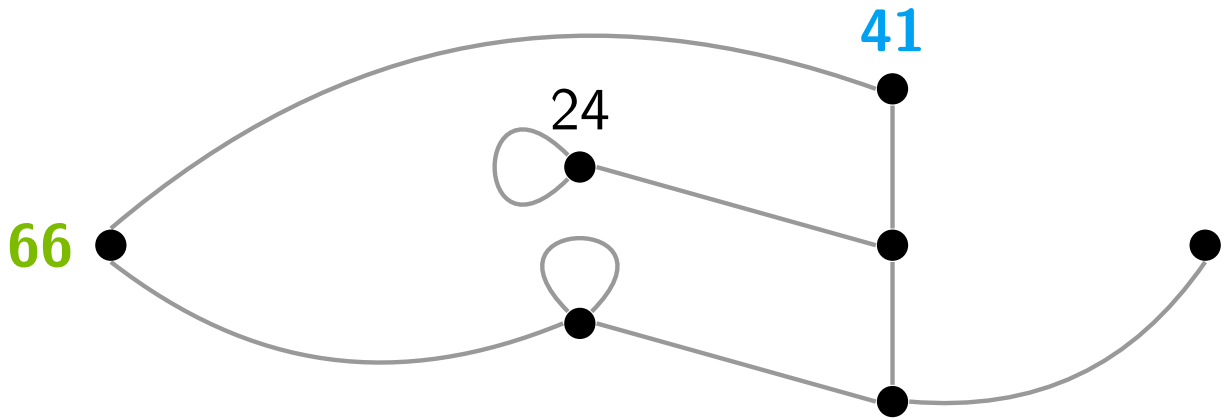

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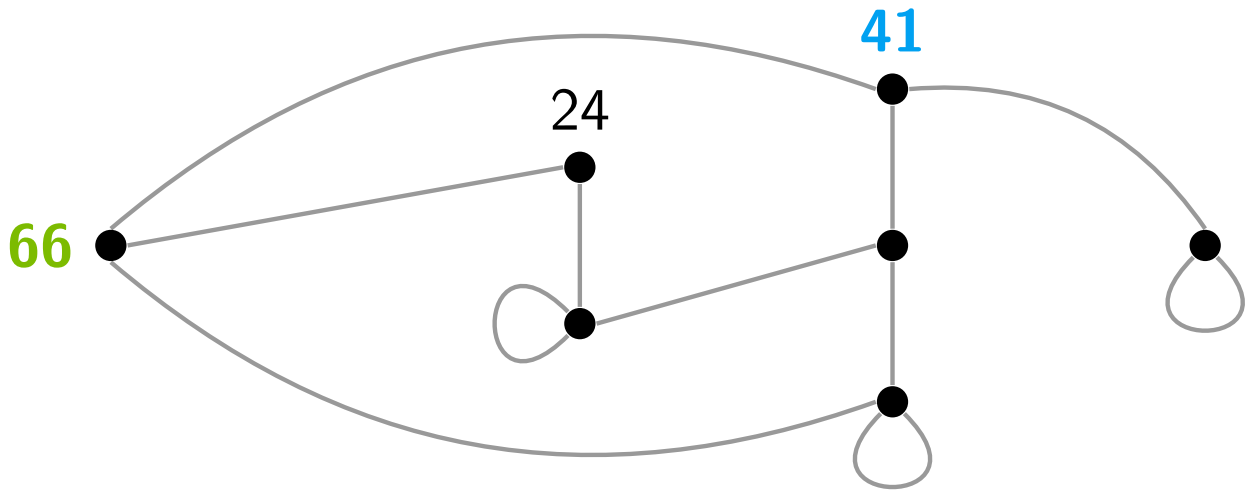

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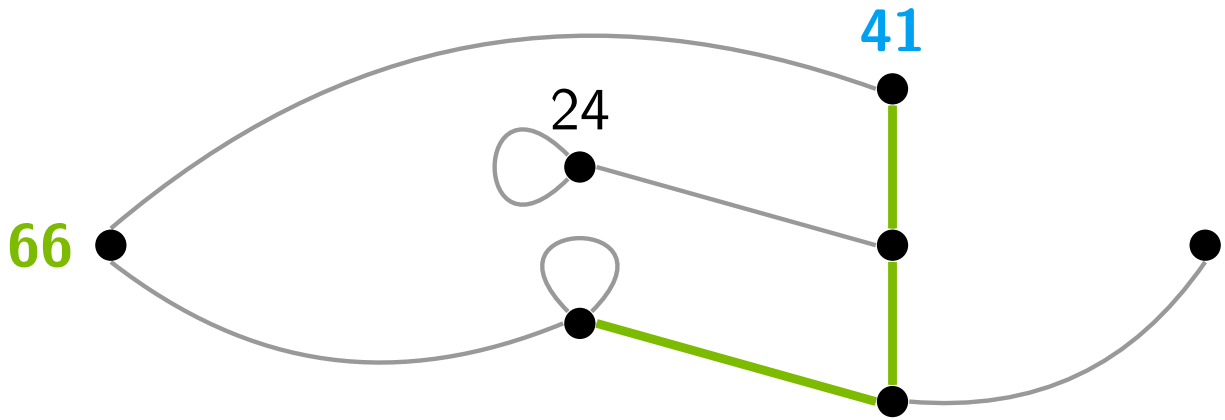

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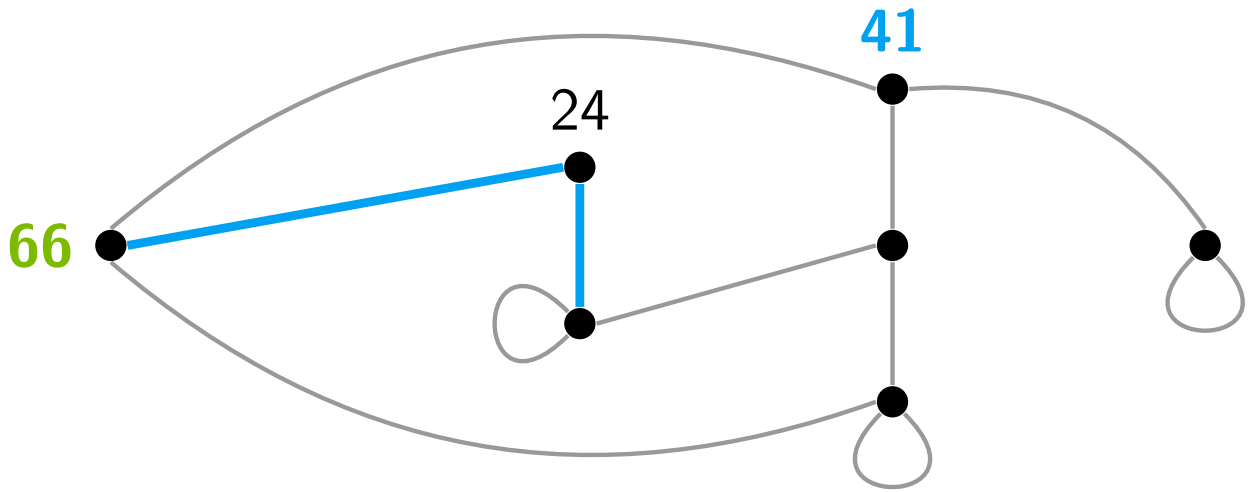

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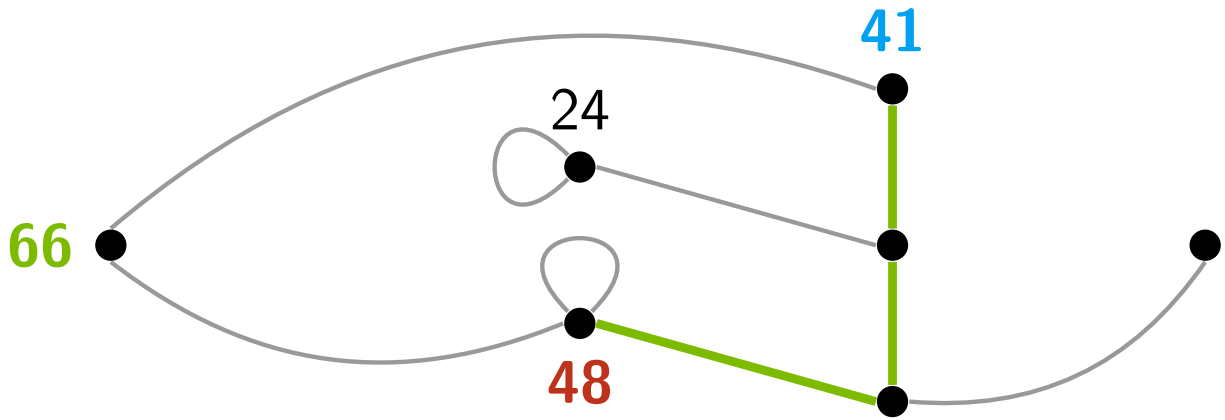

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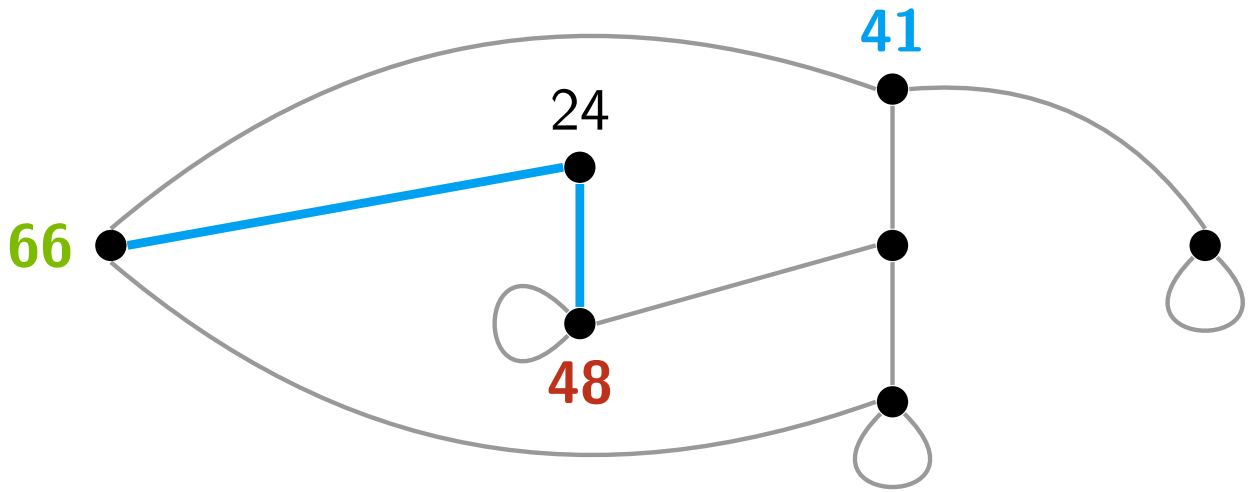
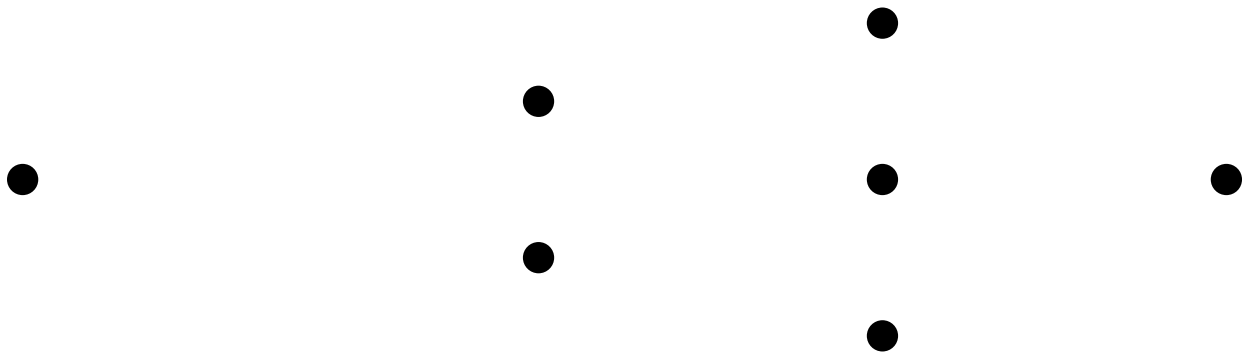


Constructing graphs and walks using isogenies



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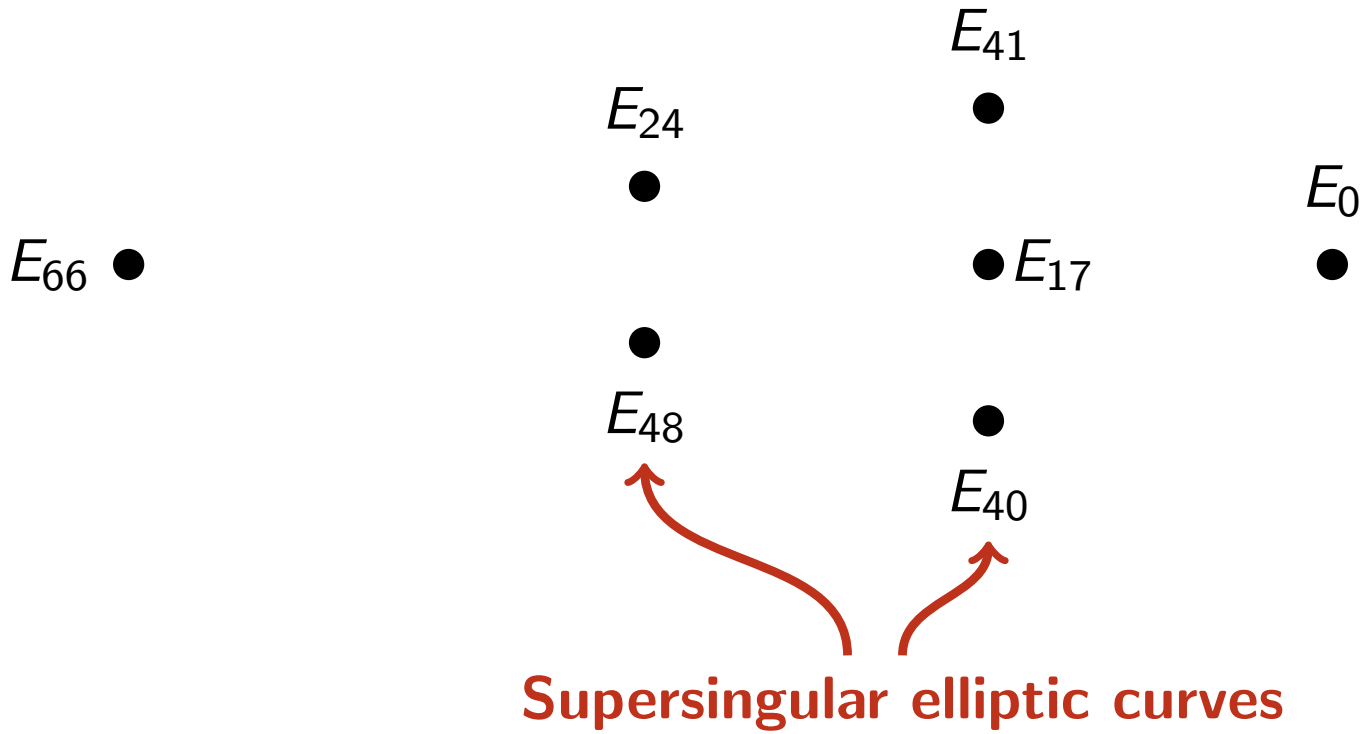

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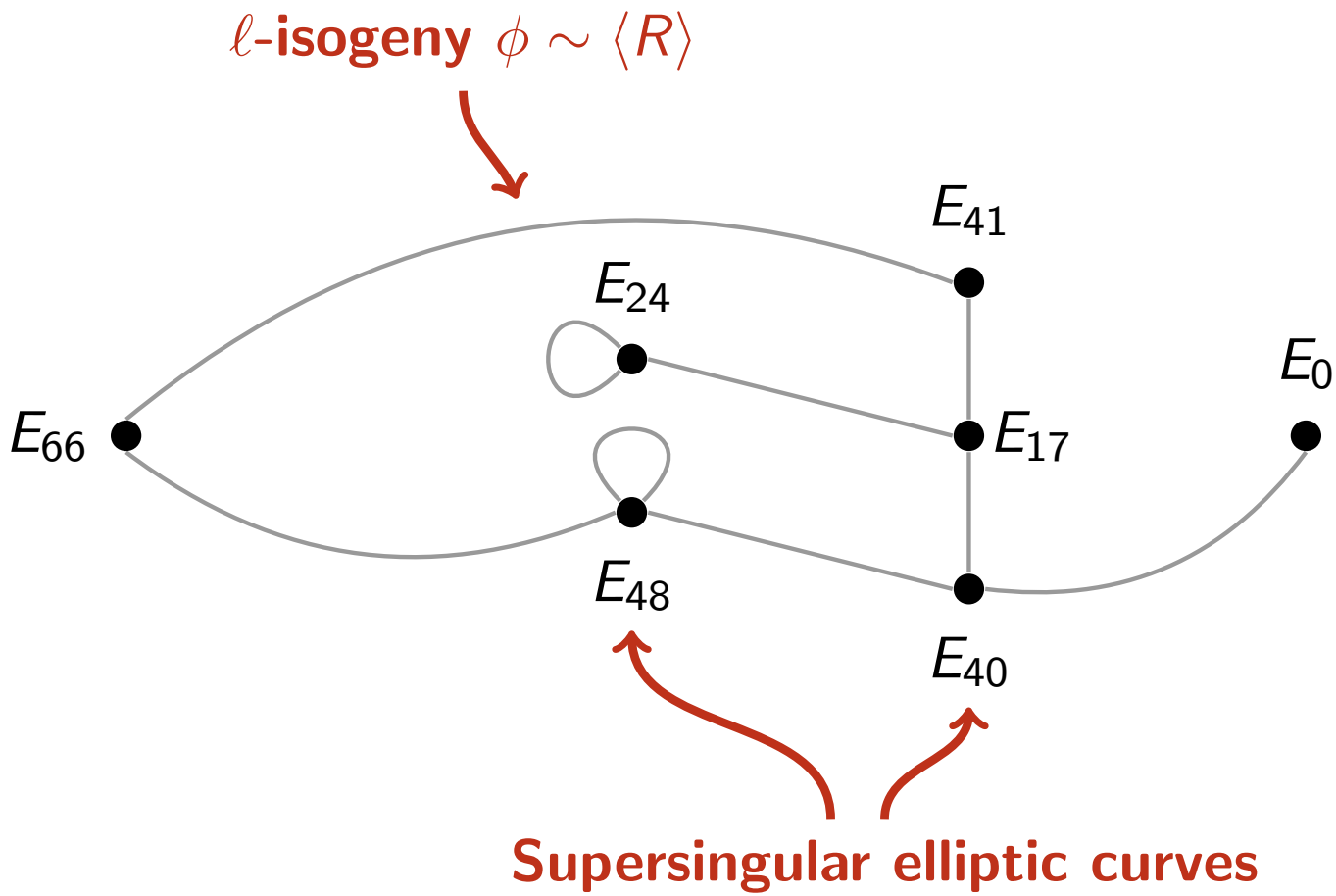

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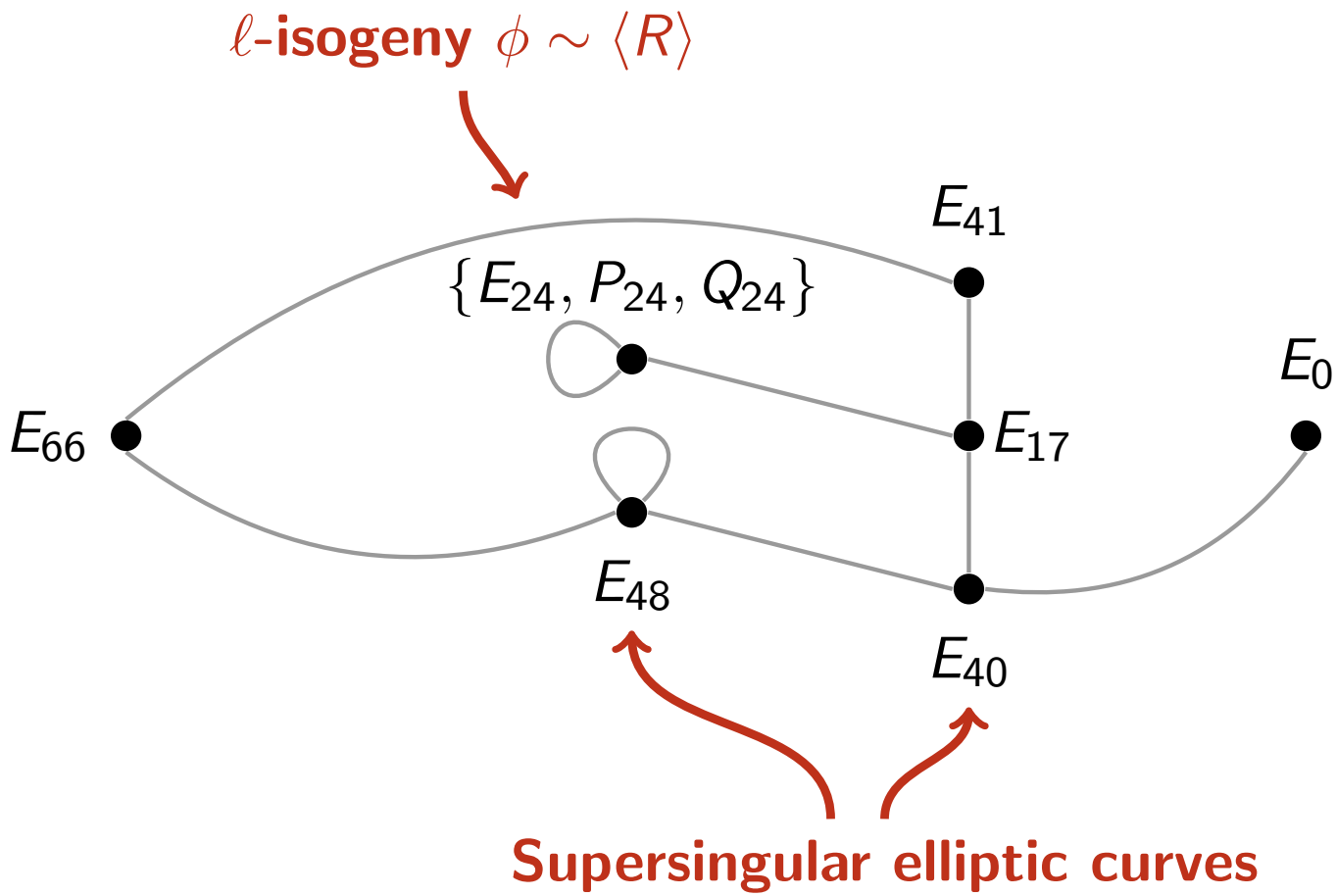

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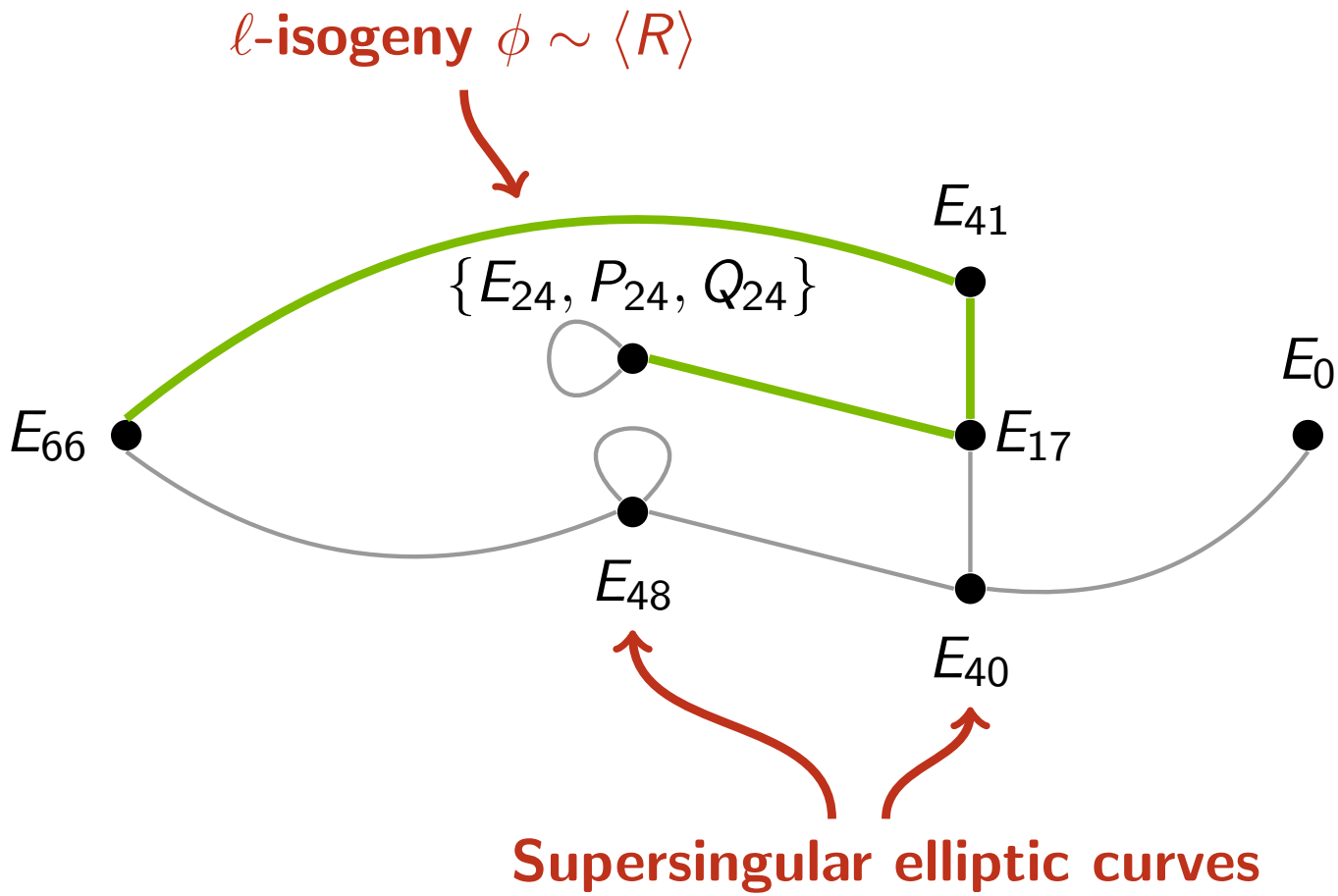

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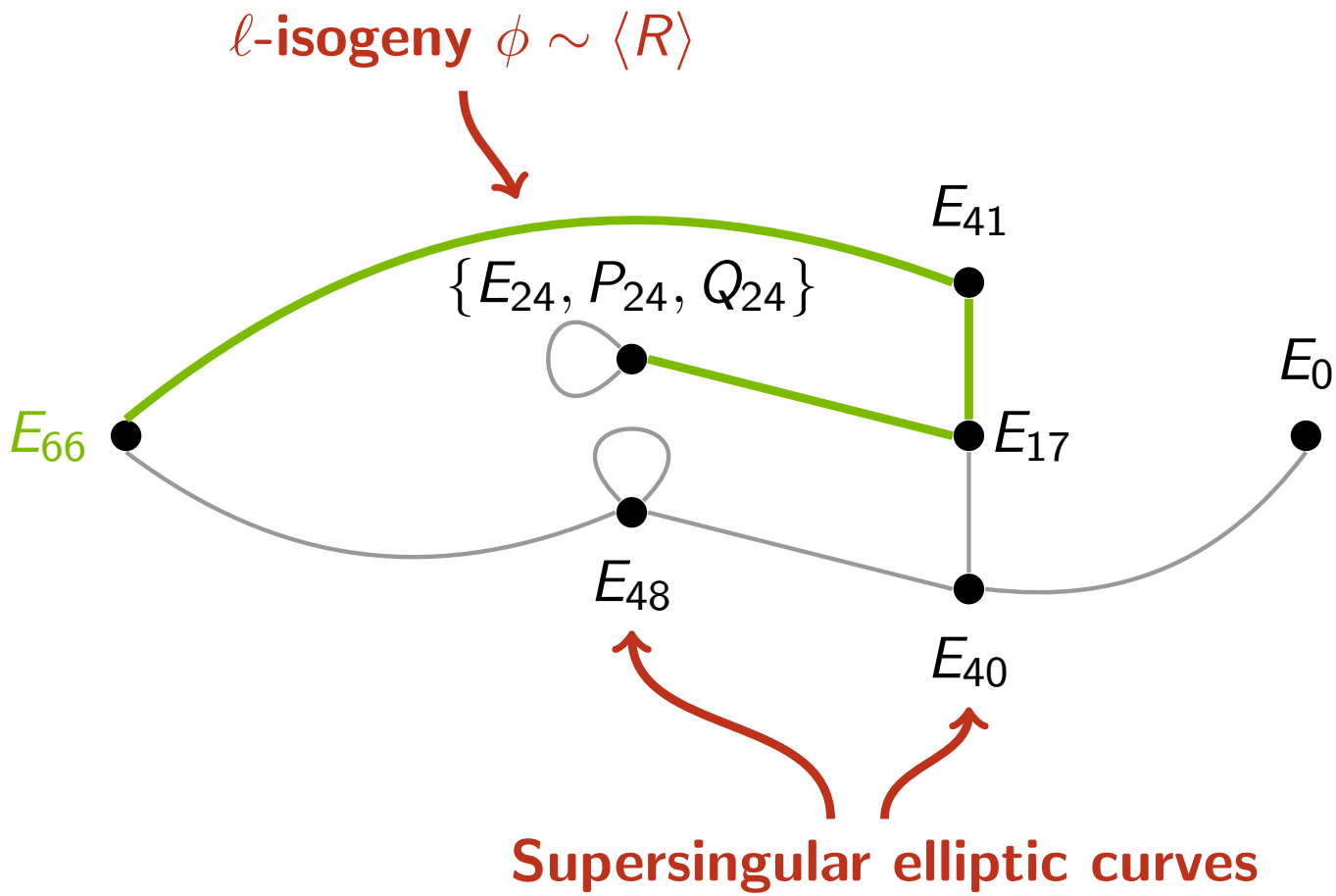

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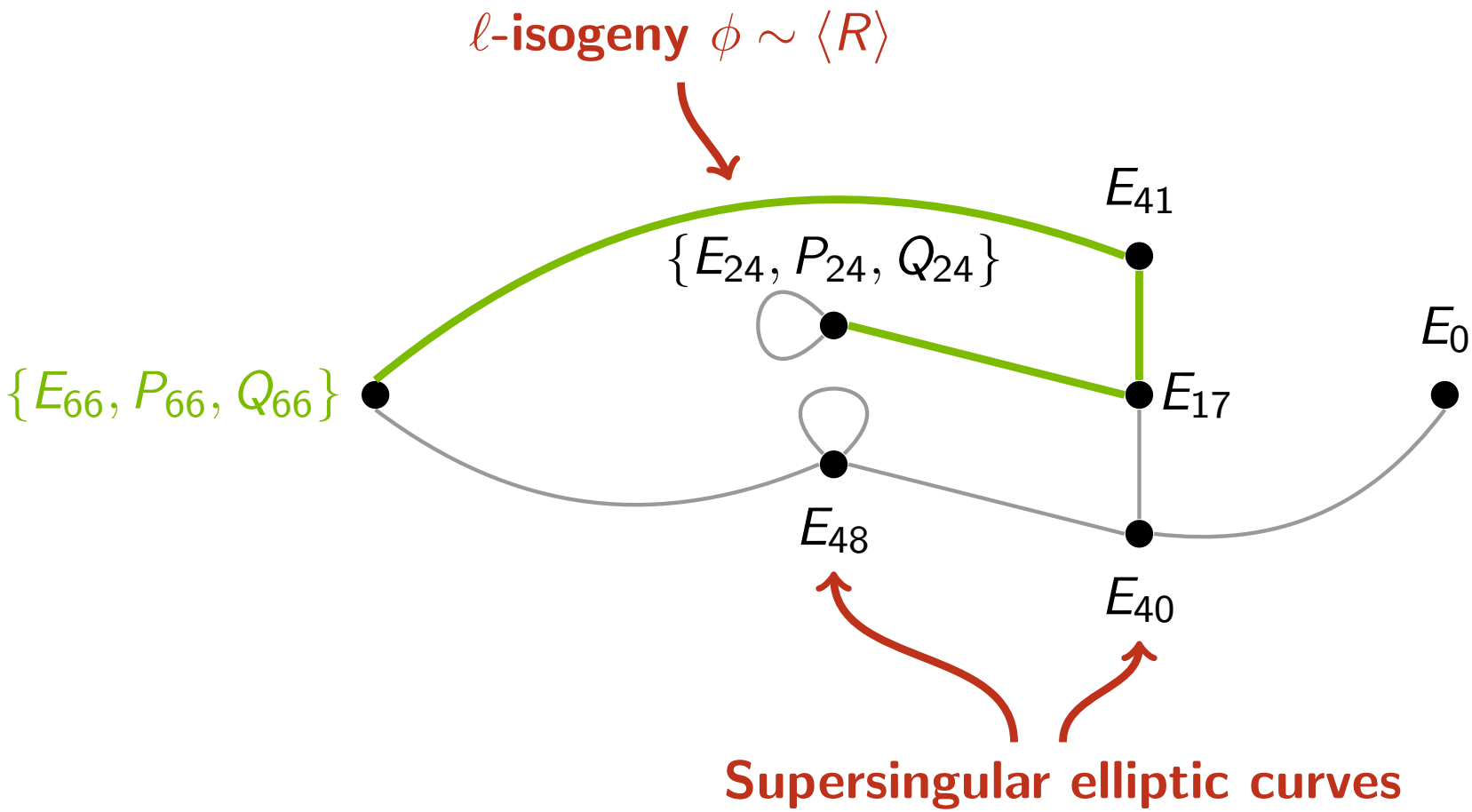

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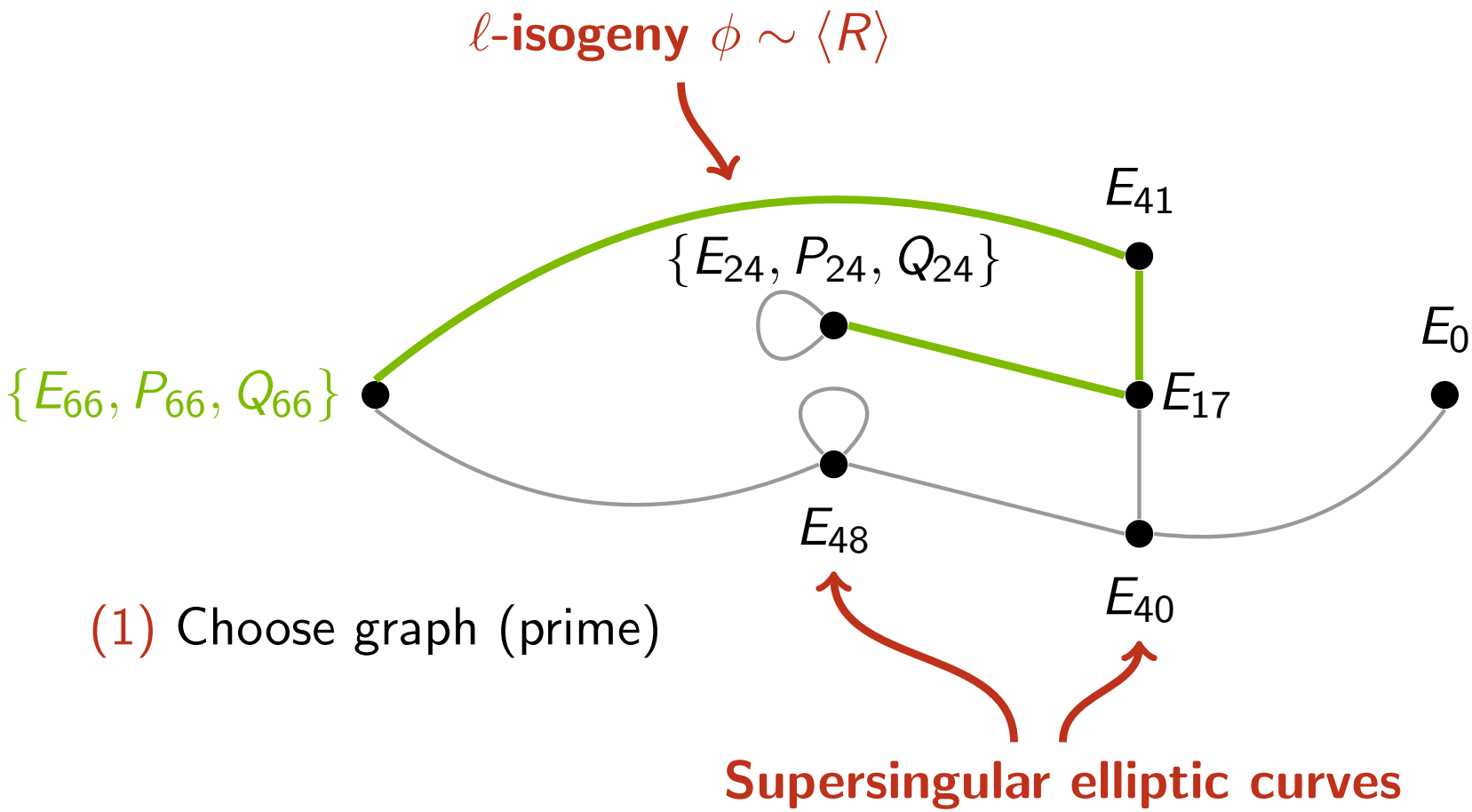

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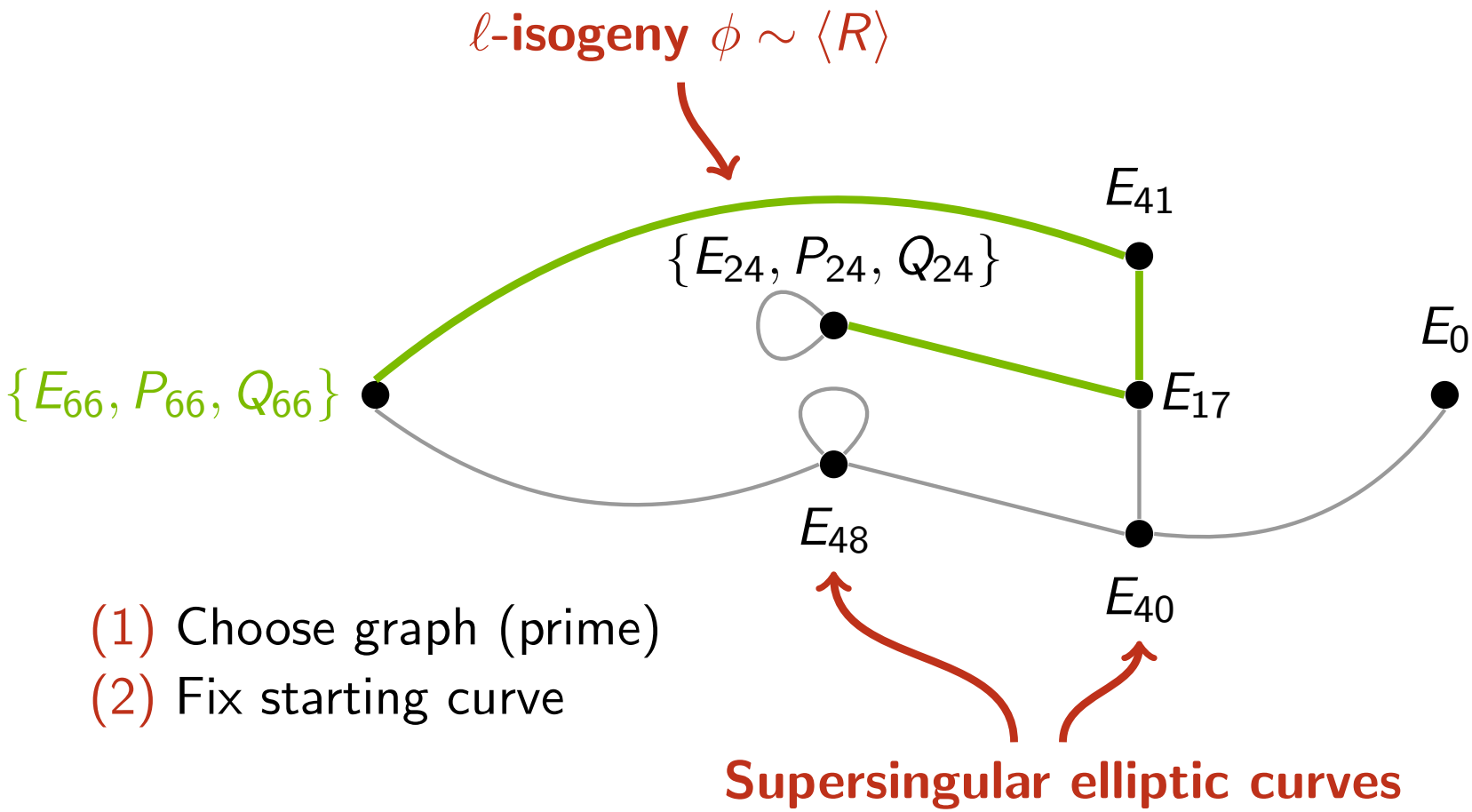

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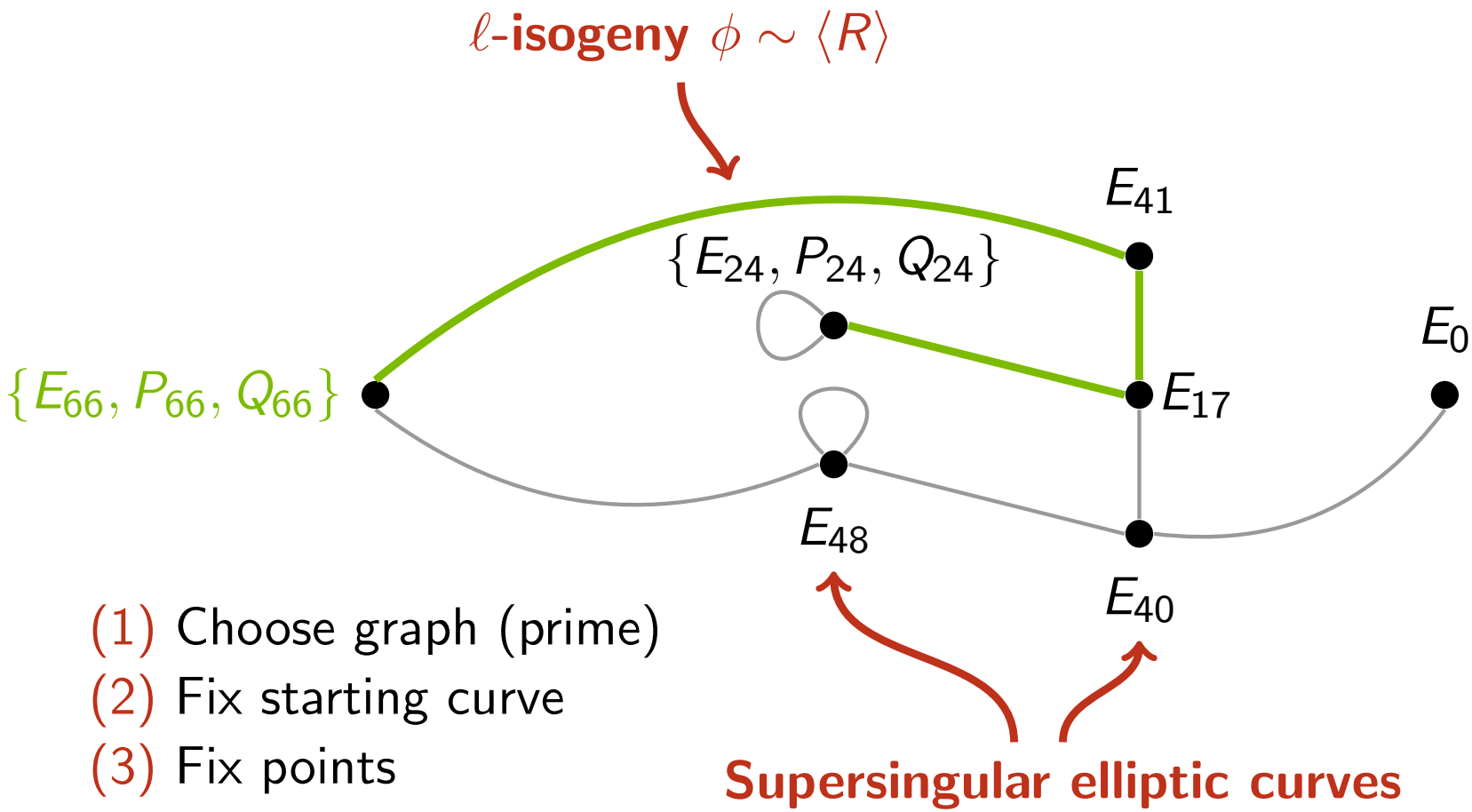

Choosing parameters

(1) Fix prime $p = 2^{e_2} \cdot 3^{e_3} - 1$

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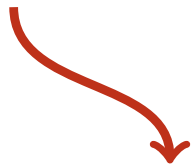
$$E_0[2^{e_2}] = \{P_2, Q_2\}, \quad E_0[3^{e_3}] = \{P_3, Q_3\}$$

Choosing parameters

Only choice to make! How large?



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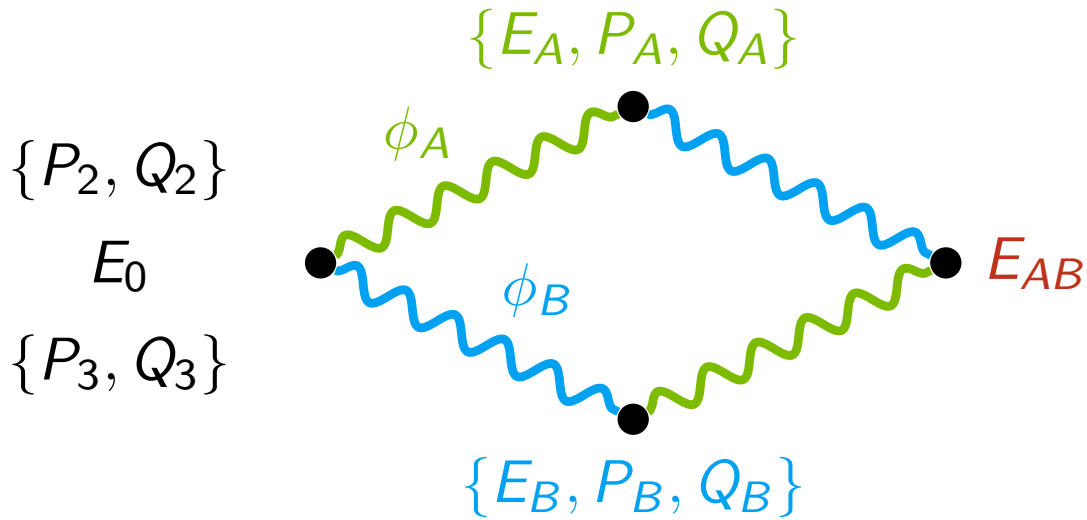
The SIDH problem



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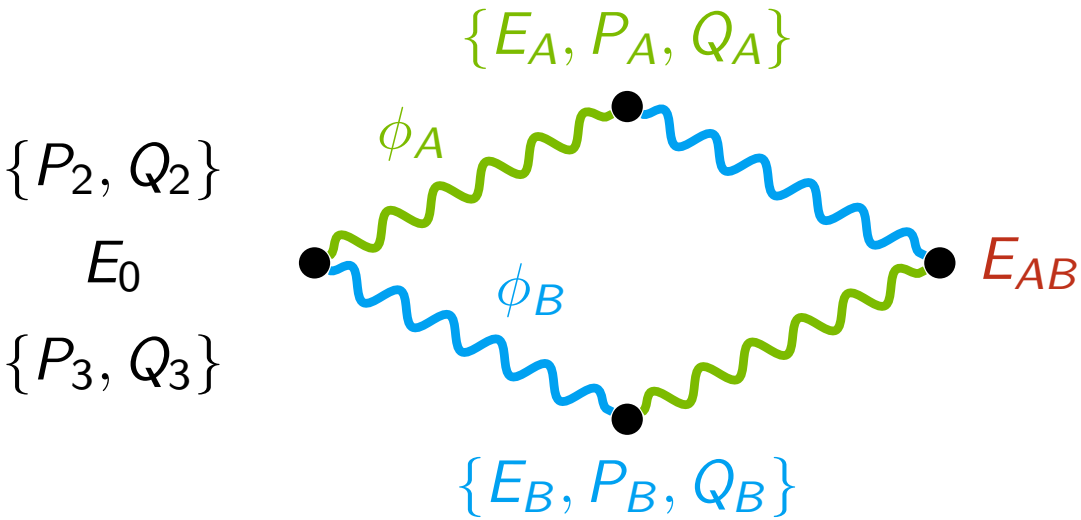
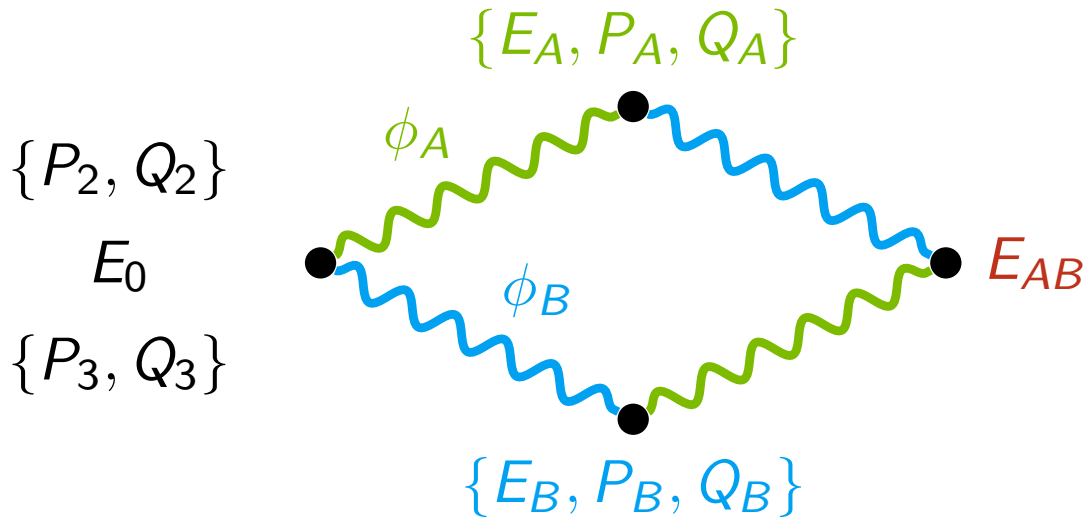


Prob. 1 (SIDH): Given $\{E_A, P_A, Q_A\}$ and $\{E_B, P_B, Q_B\}$, get E_{AB}

The SIDH problem



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Prob. 2 (SSI*): Given $\{E_A, P_A, Q_A\}$, get ϕ_A

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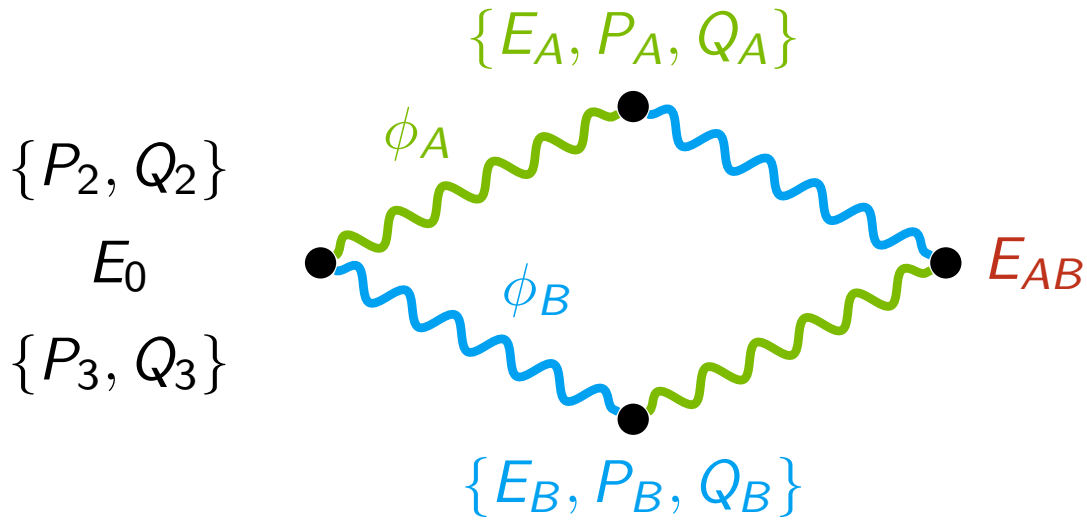

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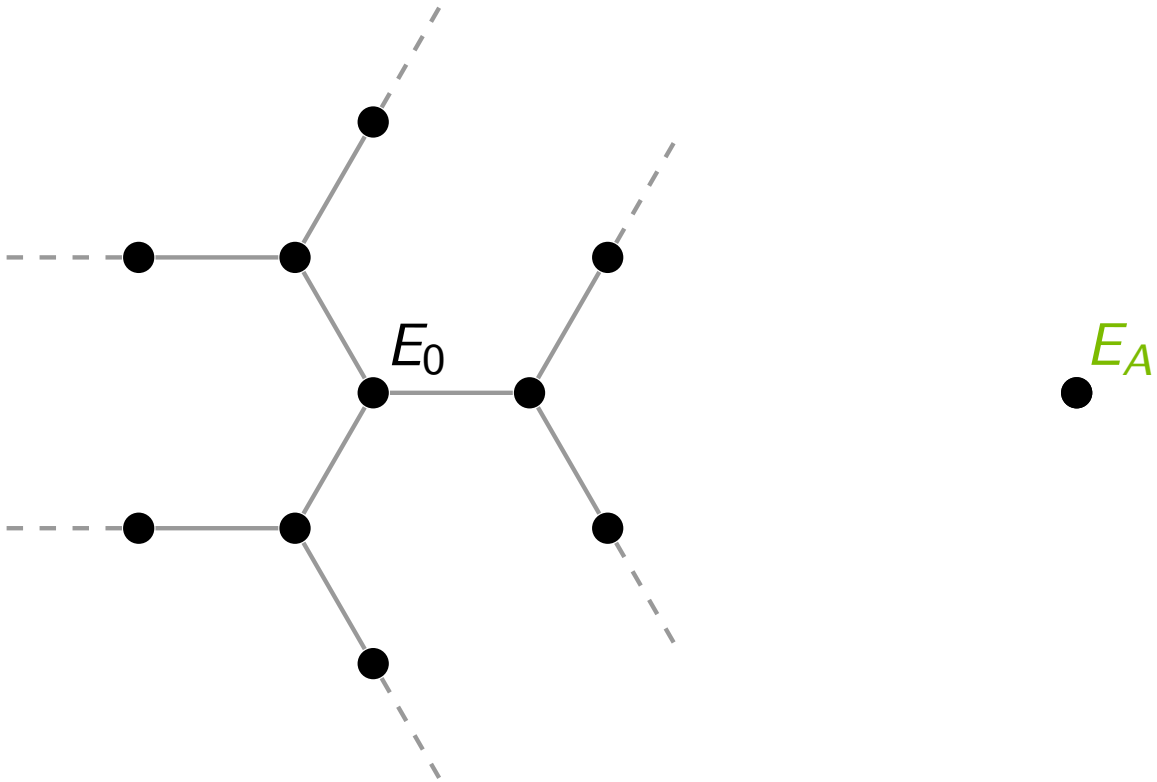
Prob. 3 (SSI): Given E_A, get ϕ_A

Solving SSI with claw finding algorithms



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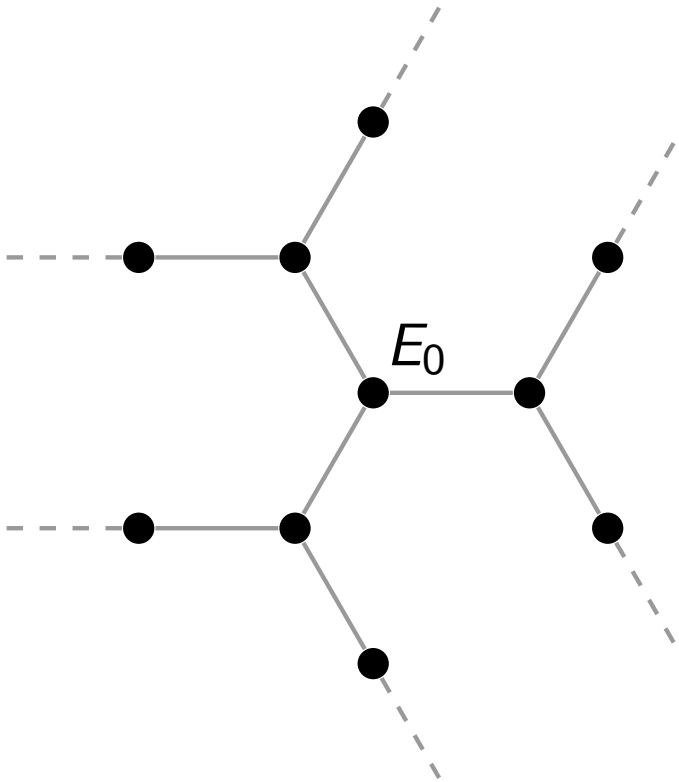

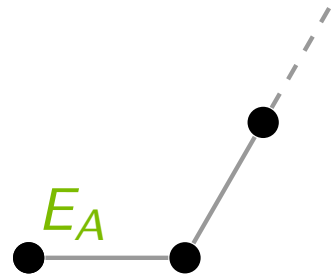

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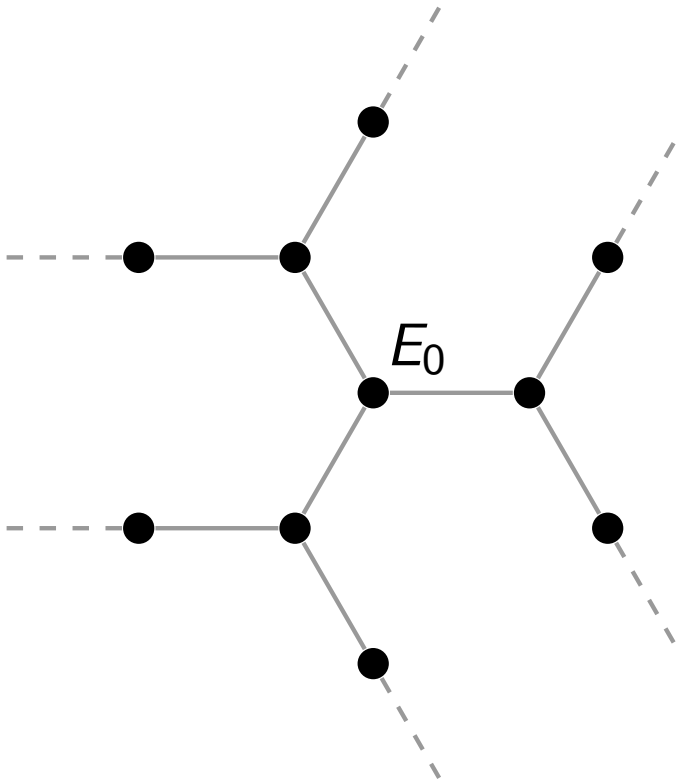

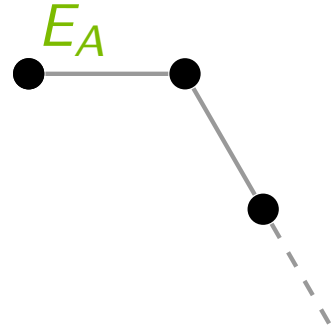

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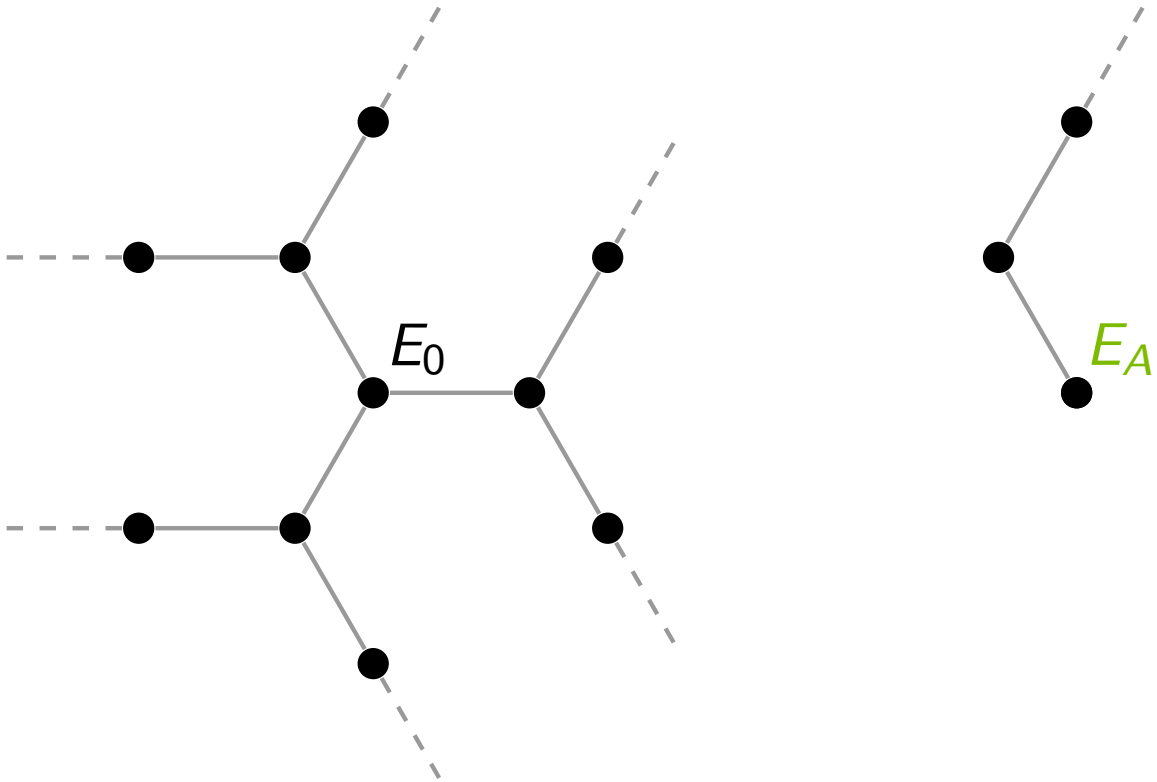

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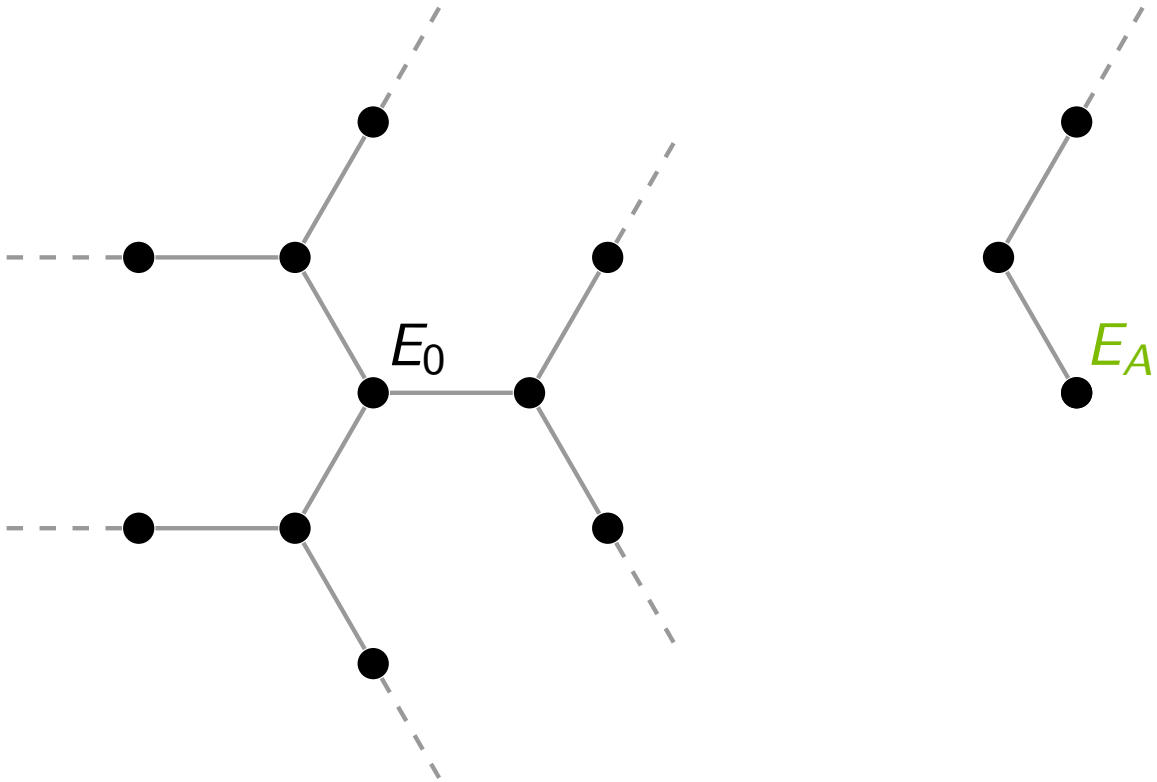

Complexity of $O(\sqrt{\deg \phi}) \sim O(\sqrt[4]{p})$ classical
$O(\sqrt[3]{\deg \phi}) \sim O(\sqrt[6]{p})$ quantum
oracle queries

Aligning security with the NIST requirements

"As secure as k-bit AES"

	Classical	Quantum
`AES128`	127	64

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`AES128`	127	64
`SIKEp503`	125	83

Aligning security with the NIST requirements

"As secure as k-bit AES"

	Classical	Quantum
`AES128`	127	64
`SIKEp503`	125	83
`AES192`	191	96
`SIKEp751`	186	124

Aligning security with the NIST requirements

"As secure as k-bit AES"

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AES128	127	64
SIKEp503	125	83
AES192	191	96
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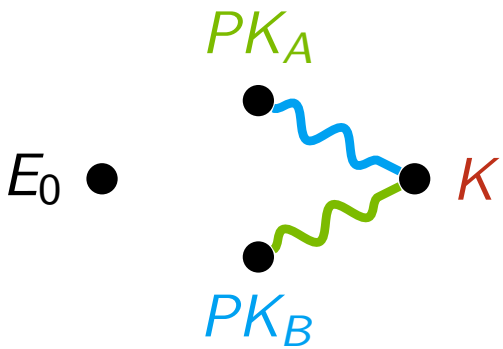
AES256	255	128
SIKEp964	238	159

What is SIKE...



SIDH

Passively secure
under (SI)CDH

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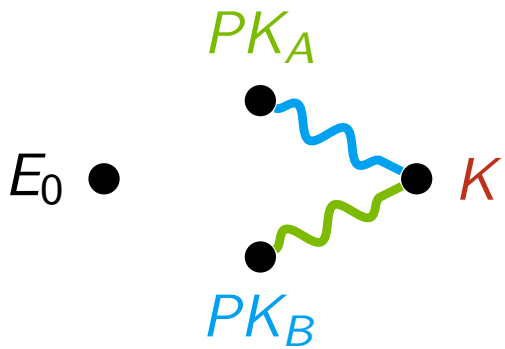

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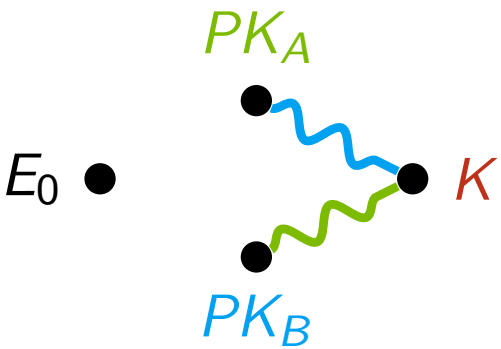

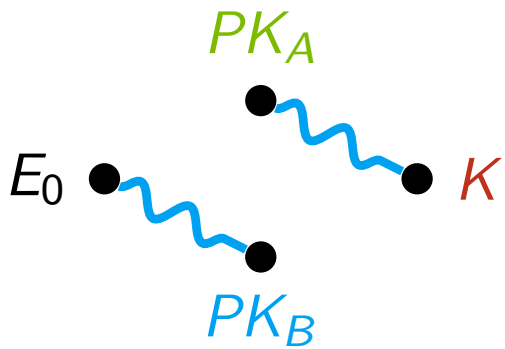

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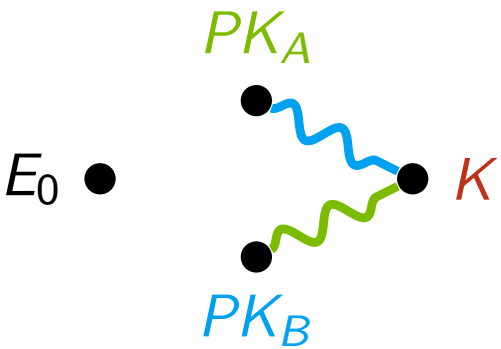

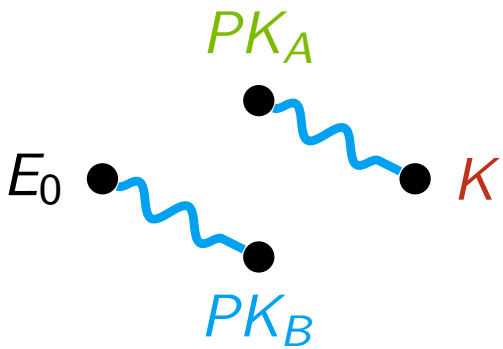

$C_0 = PK_B$

$C_1 = M \oplus F(K)$

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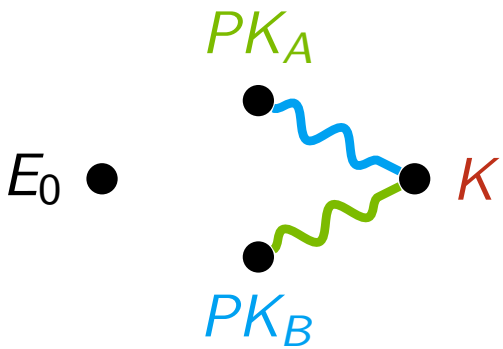

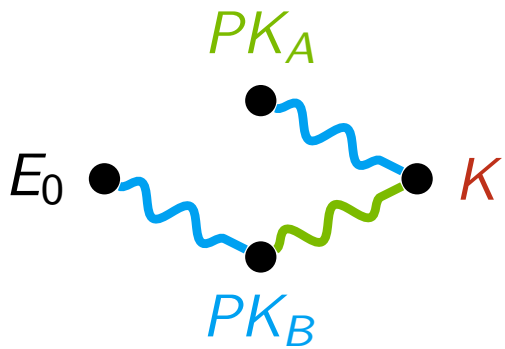

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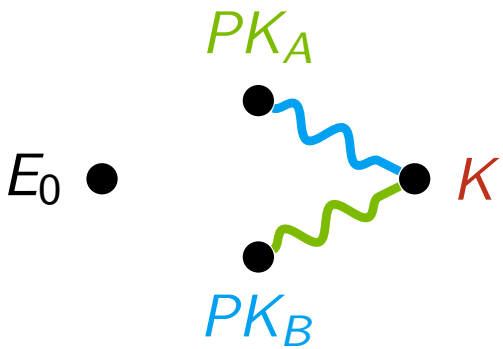

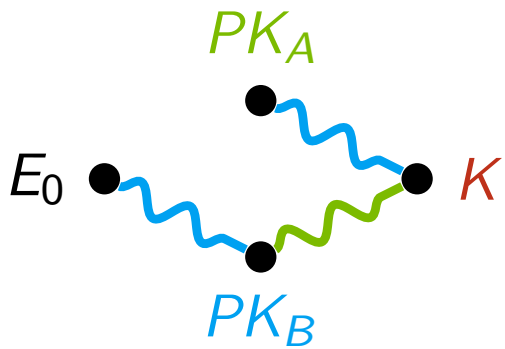

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Passively secure under (SI)CDH

ElGamal

Passively secure under (SI)CDH in ROM

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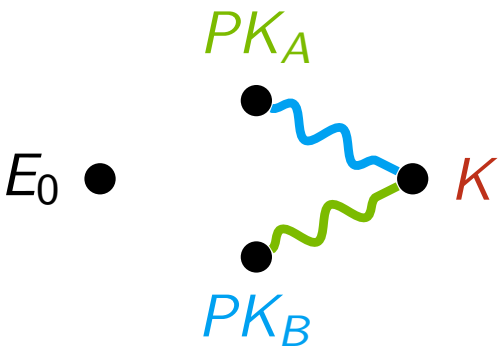

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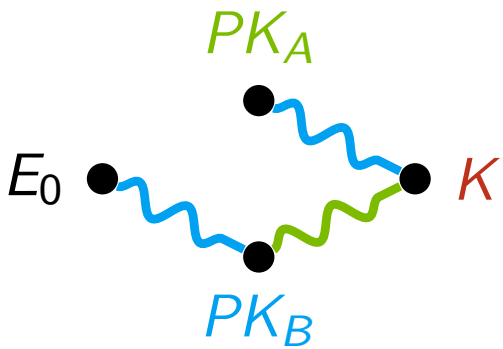

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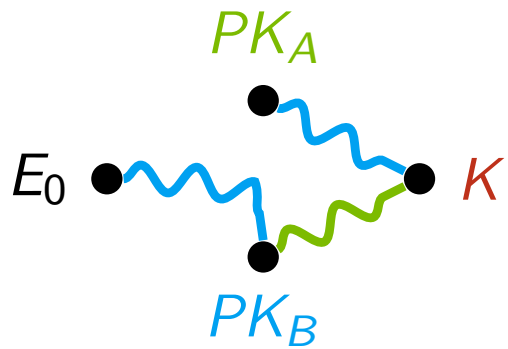
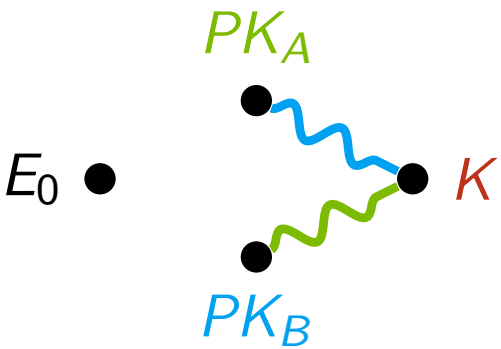
ElGamal

Passively secure under (SI)CDH in ROM



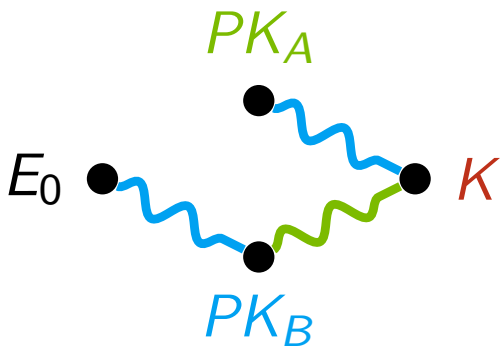
$S = H(M, C_0, C_1)$

What is SIKE...



SIDH

Passively secure under (SI)CDH

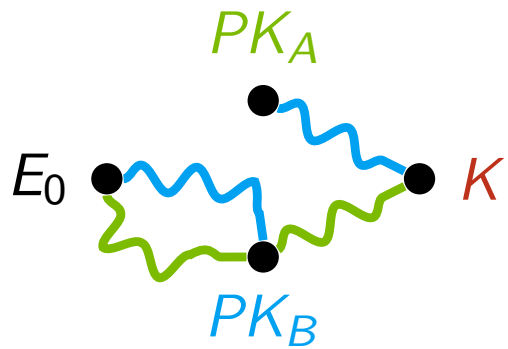


$C_0 = PK_B$

$C_1 = M \oplus F(K)$

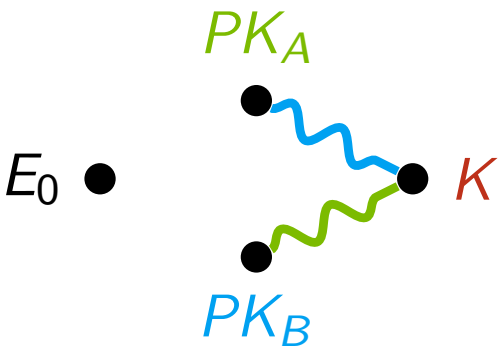
ElGamal

Passively secure under (SI)CDH in ROM



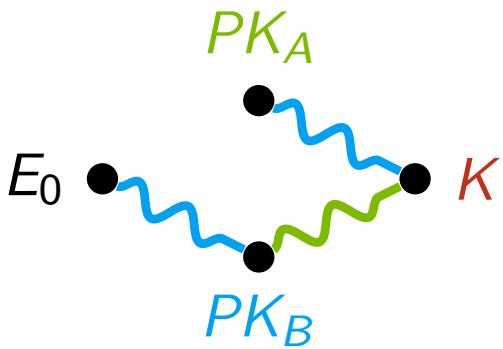
$S = H(M, C_0, C_1)$

What is SIKE...



SIDH

Passively secure under (SI)CDH

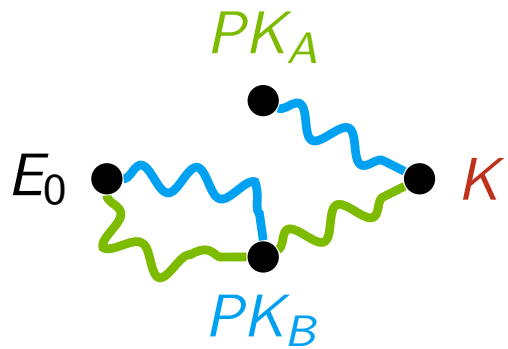


$C_0 = PK_B$

$C_1 = M \oplus F(K)$

ElGamal

Passively secure under (SI)CDH in ROM



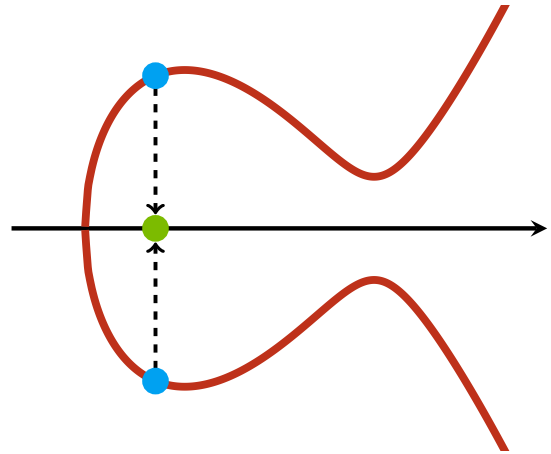
$S = H(M, C_0, C_1)$

SIKE

Actively secure under (SI)CDH in ROM

Implementation choices: curve model

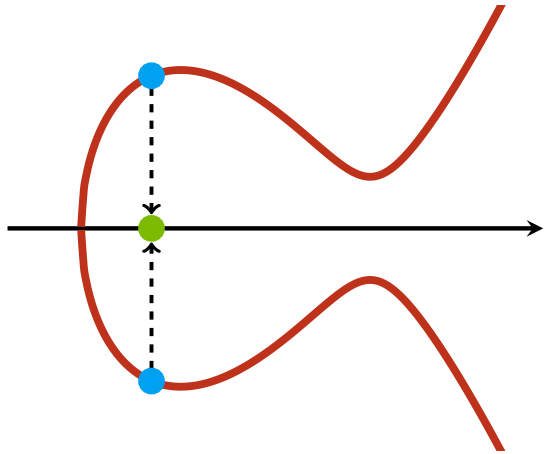
(1) Model choice: Montgomery



Implementation choices: curve model

(1) Model choice: Montgomery

(2) Only x-coordinates needed



$$E_0 \xrightarrow{\phi} E_1$$

$$(x, —) \longmapsto (f(x), —)$$

$$\deg(f) = \deg(\phi)$$

Computing isogenies

(3) Tree-based isogeny computation

$$(E_0, P_{00})$$

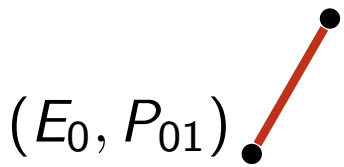
$$\bullet$$

Order of P_{00} is ℓ^e

$$\implies \deg(\phi_{00}) = \ell^e$$

Computing isogenies

(3) Tree-based isogeny computation



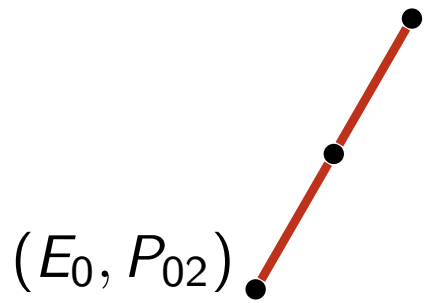
$P_{01} = [\ell]P_{00}$

Order of P_{01} is ℓ^{e-1}

$\implies \deg(\phi_{01}) = \ell^{e-1}$

Computing isogenies

(3) Tree-based isogeny computation



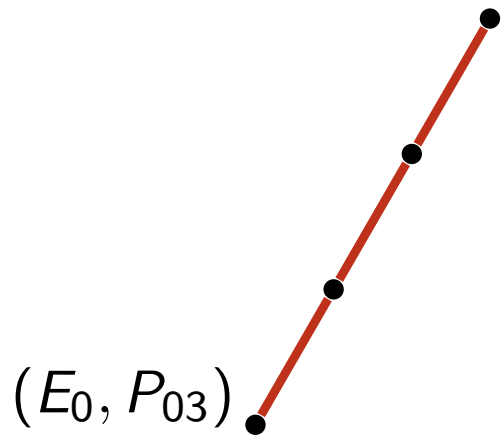
$P_{02} = [\ell^2]P_{00}$

Order of P_{02} is ℓ^{e-2}

$\implies \deg(\phi_{02}) = \ell^{e-2}$

Computing isogenies

(3) Tree-based isogeny computation



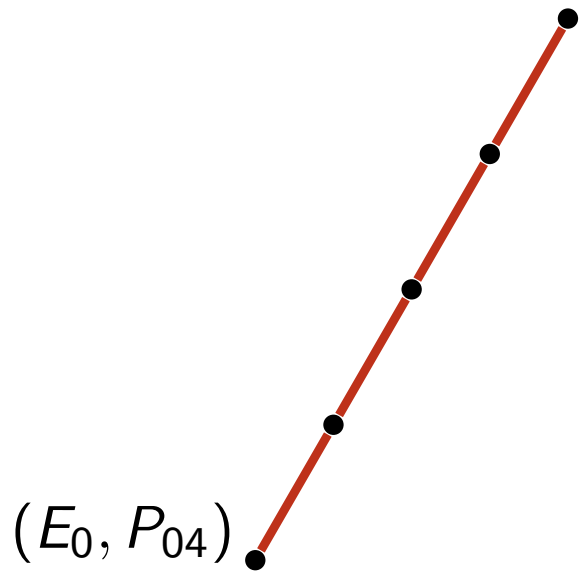
$P_{03} = [\ell^3]P_{00}$

Order of P_{03} is ℓ^{e-3}

$\implies \deg(\phi_{03}) = \ell^{e-3}$

Computing isogenies

(3) Tree-based isogeny computation



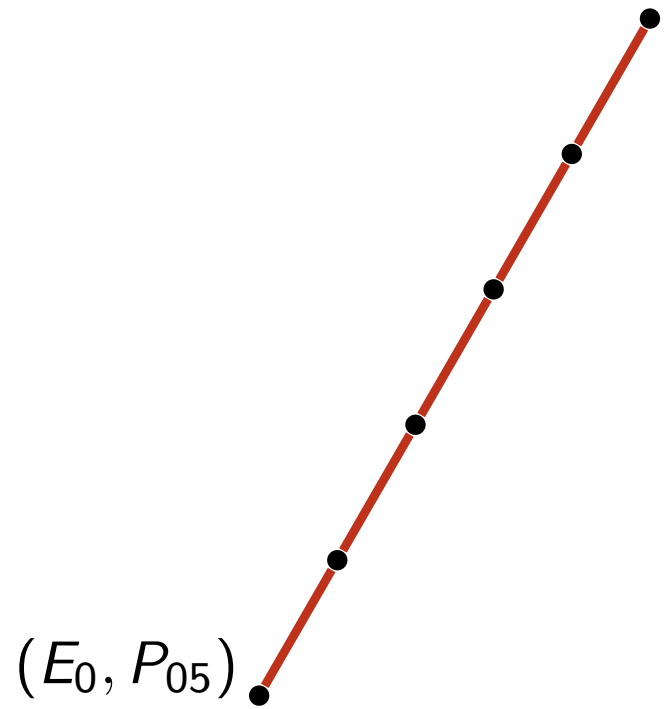
$P_{04} = [\ell^4]P_{00}$

Order of P_{04} is ℓ^{e-4}

$\implies \deg(\phi_{04}) = \ell^{e-4}$

Computing isogenies

(3) Tree-based isogeny computation



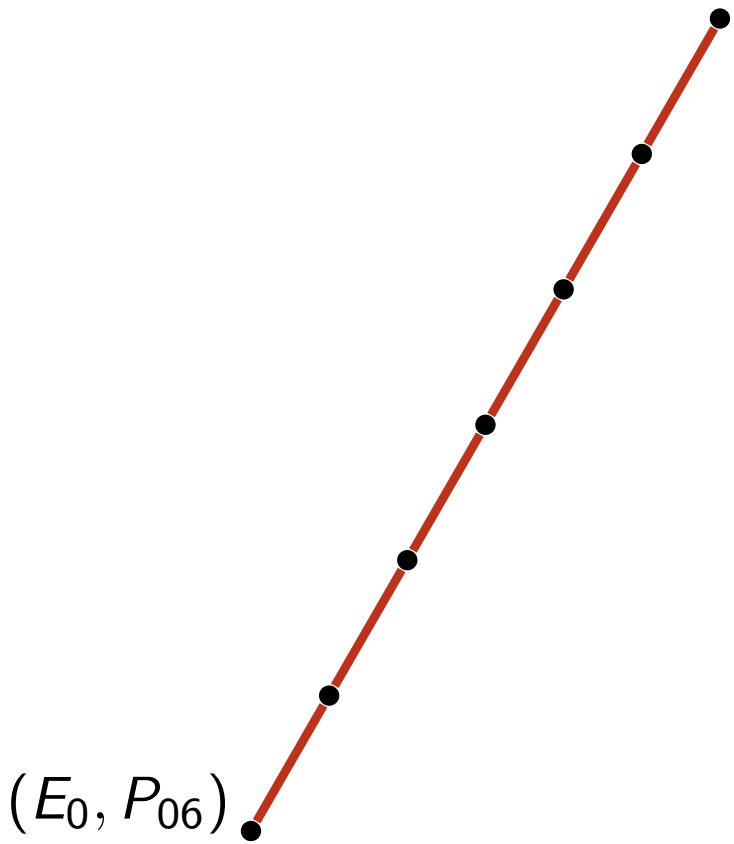
$P_{05} = [\ell^5]P_{00}$

Order of P_{05} is ℓ^{e-5}

$\implies \deg(\phi_{05}) = \ell^{e-5}$

Computing isogenies

(3) Tree-based isogeny computation



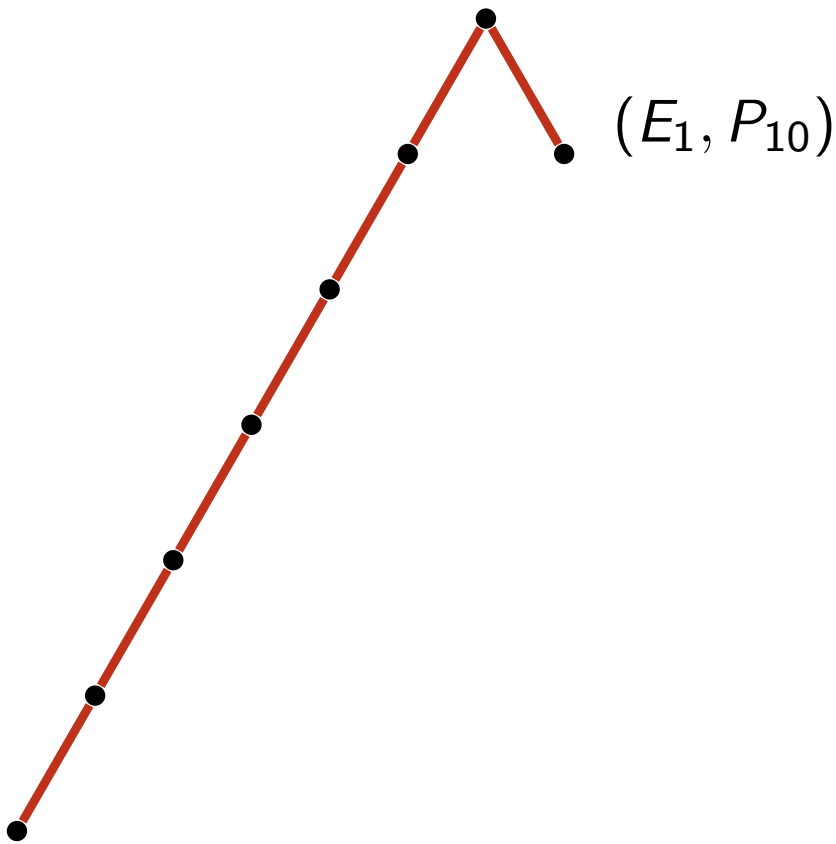
$P_{06} = [\ell^6]P_{00}$

Order of P_{06} is ℓ^{e-6}

$\implies \deg(\phi_{06}) = \ell^{e-6}$

Computing isogenies

(3) Tree-based isogeny computation



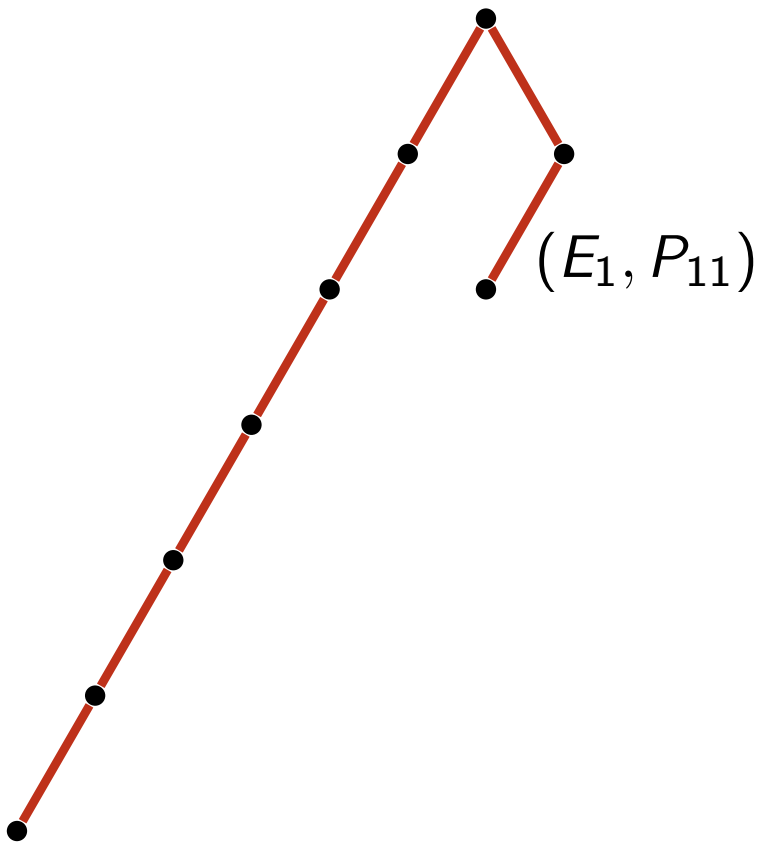
$P_{10} = \phi_{00}(P_{00})$

Order of P_{10} is ℓ^{e-1}

$\implies \deg(\phi_{10}) = \ell^{e-1}$

Computing isogenies

(3) Tree-based isogeny computation



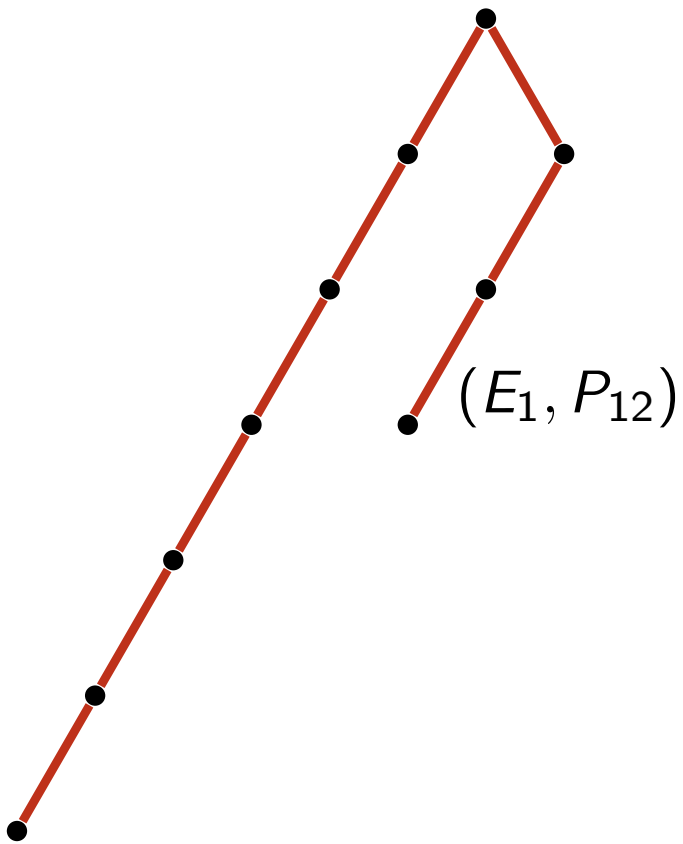
$P_{11} = [\ell]P_{10}$

Order of P_{11} is ℓ^{e-2}

$\implies \deg(\phi_{11}) = \ell^{e-2}$

Computing isogenies

(3) Tree-based isogeny computation



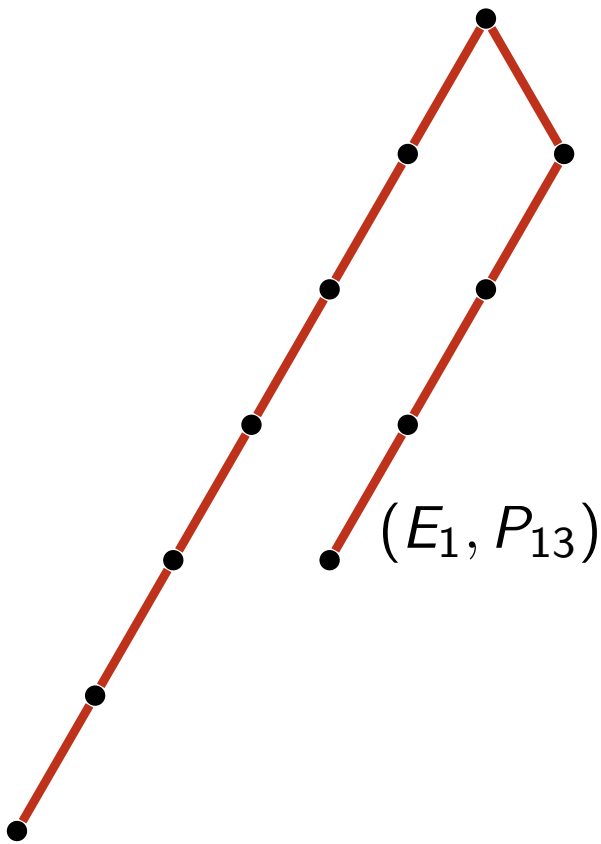
$P_{12} = [\ell^2]P_{10}$

Order of P_{12} is ℓ^{e-3}

$\implies \deg(\phi_{12}) = \ell^{e-3}$

Computing isogenies

(3) Tree-based isogeny computation



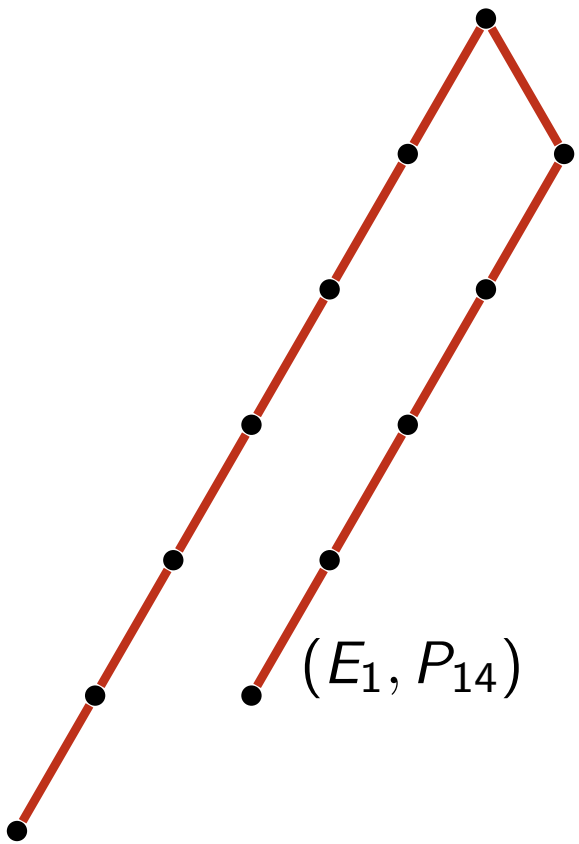
$P_{13} = [\ell^3]P_{10}$

Order of P_{13} is ℓ^{e-4}

$\implies \deg(\phi_{13}) = \ell^{e-4}$

Computing isogenies

(3) Tree-based isogeny computation



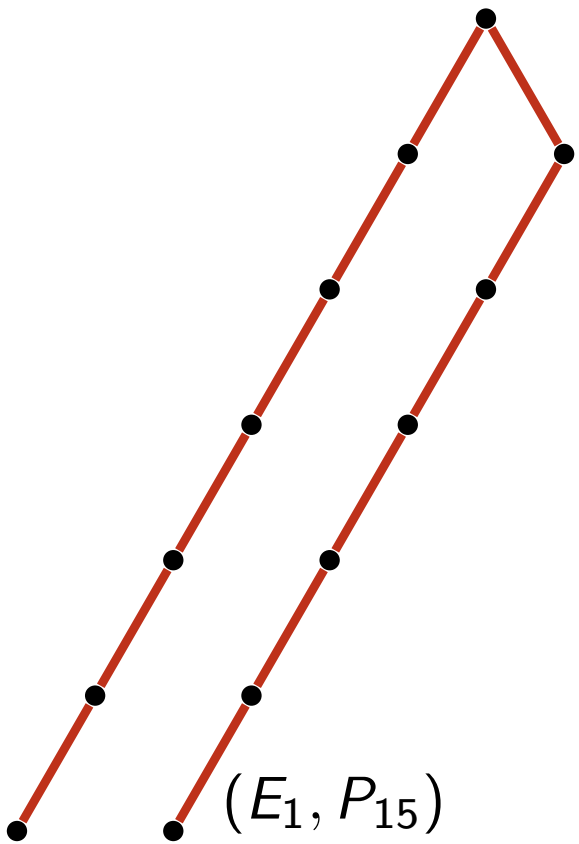
$P_{14} = [\ell^4]P_{10}$

Order of P_{14} is ℓ^{e-5}

$\implies \deg(\phi_{14}) = \ell^{e-5}$

Computing isogenies

(3) Tree-based isogeny computation



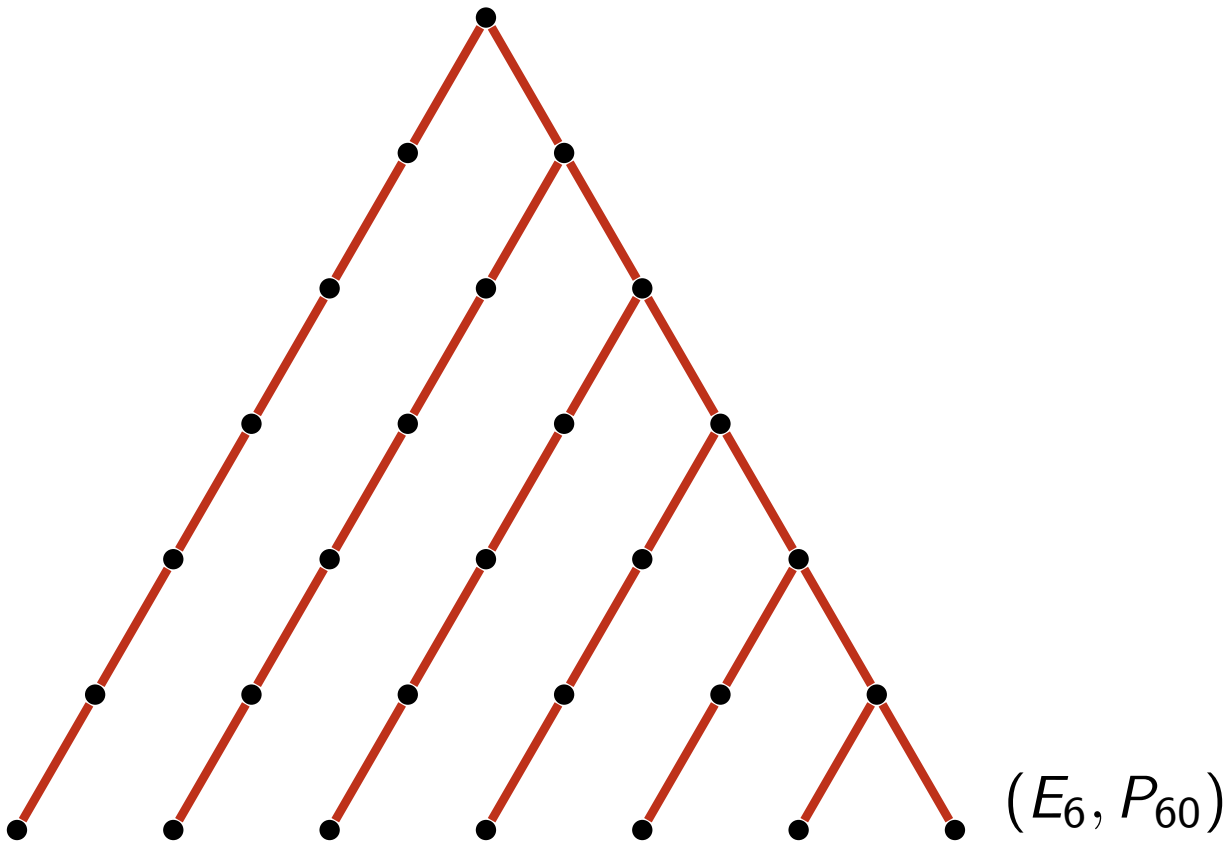
$P_{15} = [\ell^5]P_{10}$

Order of P_{15} is ℓ^{e-6}

$\implies \deg(\phi_{15}) = \ell^{e-6}$

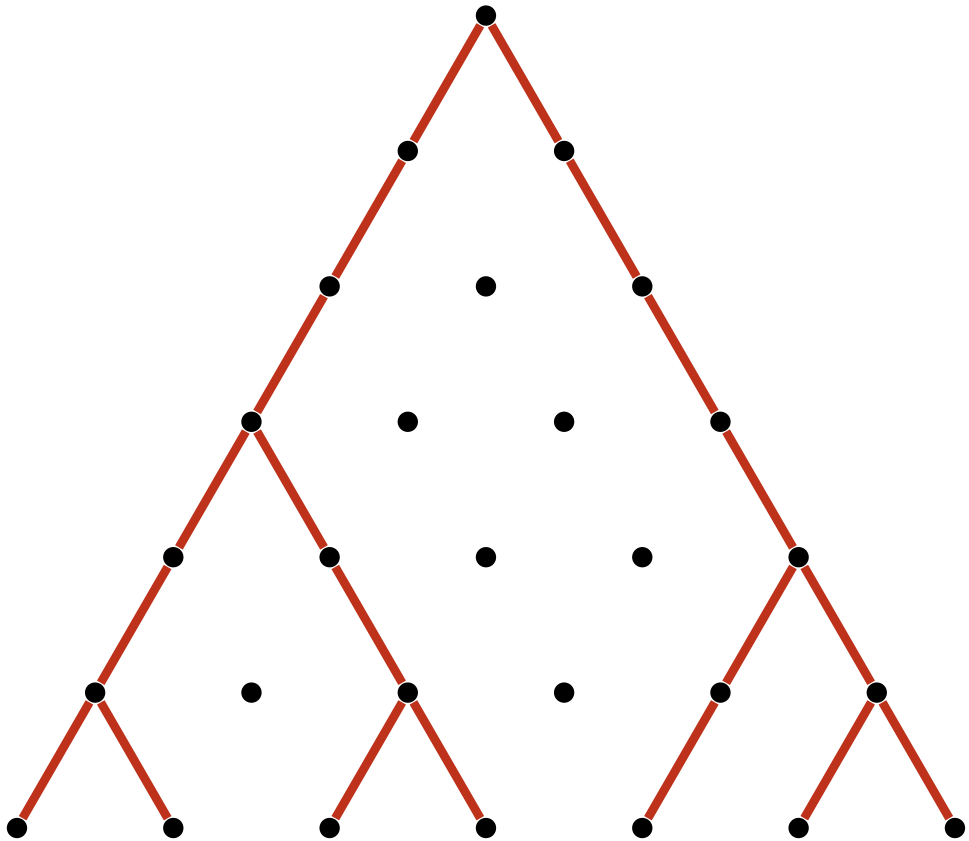
Computing isogenies

(3) Tree-based isogeny computation



Computing isogenies

(3) Tree-based isogeny computation



Where to begin

(4) Starting curve $E_0 : y^2 = x^3 + x$ with $j = 1728$

$\Longrightarrow$ Know things about $\mathrm{End}(E_0)$, could help attacks..[1]

$\Longrightarrow$ Defined over $\mathbb{F}_p \subset \mathbb{F}_{p^2}$

$\Longrightarrow$ Attack $O(\sqrt{p})$ (with low memory[2])

$\Longrightarrow$ No better way to obtain a random starting curve?

[1] Petit '17

[2] Delfs, Galbraith '13

Other implementation choices

(5) No public-key compression

Other implementation choices

(5) No public-key compression

(6) Sym. functions cSHAKE256

Final numbers

	Speed (ms)	PK (Kbytes)
RSA 3072	4.6	0.8
NIST P-256	1.4	0.1
Kyber	0.07	1.2
FrodoKEM	1.2–2.3	9.5 – 15.4
SIKEp503	10.1	0.4
SIKEp751	30.5	0.6
SIDHp503	10.3	0.4
SIDHp751	31.5	0.6

(Numbers from Patrick Longa's RWC'18 talk, measured on different platforms..)

Thanks

All details can be found at:
https://csrc.nist.gov/CSRC/media/Projects/Post-Quantum-Cryptography/documents/round-1/submissions/SIKE.zip

All authors:
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