Public-key Compression for SIKE

Michael Naehrig   Joost Renes

Microsoft Research   Radboud University

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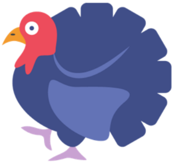
Post-Quantum Cryptography

(generic post-quantum crypto intro...)

Ostrich — Turkey

CECPQ2 = HRSS + X25519

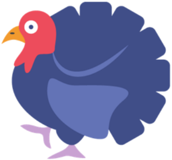


CECPQ2b = SIKE + X25519

https://blog.cloudflare.com/the-tls-post-quantum-experiment/

Ostrich — Turkey — Chicken

CECPQ2 = HRSS + X25519



CECPQ2b = SIKE + X25519

CECPQ2c = SIKEc + X25519

https://blog.cloudflare.com/the-tls-post-quantum-experiment/

Isogenies: Rational Maps between Elliptic Curves

$\mathbb{Z}$

Polynomial

Described by **coefficients**

$$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n = \sum_{i=0}^{n} c_i x^i$$

$\mathbb{Z}$

Isogenies: Rational Maps between Elliptic Curves

$\mathbb{Z}$

Polynomial

Described by **roots**

$$f(x) = c \cdot \prod_{i=0}^{n} (x - a_i)$$

$\mathbb{Z}$

Isogenies: Rational Maps between Elliptic Curves

$\mathbb{Q}$

Rational map

Described by **roots** of f and g

$$f(x)/g(x) = \prod_{i=0}^{n}(x - a_i) / \prod_{i=0}^{n}(x - b_i)$$

$\mathbb{Q}$

Isogenies: Rational Maps between Elliptic Curves

$\overline{\mathbb{F}}_p$

Rational map

Described by **roots** of f and g

$$f(x)/g(x) = \prod_{i=0}^{n}(x - a_i) / \prod_{i=0}^{n}(x - b_i)$$

$\overline{\mathbb{F}}_p$

Isogenies: Rational Maps between Elliptic Curves

$$\overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p$$

Rational map

$$\Big(f(x,y)/g(x,y), h(x,y)/k(x,y) \Big)$$

$$\downarrow$$

$$\overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p$$

Isogenies: Rational Maps between Elliptic Curves

$$\{ (x, y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + A \cdot x^2 + x \} \cup \{\infty\}$$

Rational map

$$\Big(f(x,y)/g(x,y), h(x,y)/k(x,y) \Big)$$

$$\downarrow$$

$$\{ (x, y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + \overline{A} \cdot x^2 + x \} \cup \{\infty\}$$

Isogenies: Rational Maps between Elliptic Curves

$$\{\ (x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + A \cdot x^2 + x\ \} \cup \{\infty\}$$

Isogeny $(\infty \mapsto \infty)$

$$\Big(f(x)/g(x), y \cdot h(x)/k(x) \Big)$$

$$\{\ (x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + \overline{A} \cdot x^2 + x\ \} \cup \{\infty\}$$

Isogenies: Rational Maps between Elliptic Curves

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Isogeny ($\infty \mapsto \infty$)

Described by **roots** of **g**

$$\left(f(x)/g(x), y \cdot h(x)/k(x) \right)$$

$$\{\,(x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + \overline{A} \cdot x^2 + x\,\} \cup \{\infty\}$$

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Isogeny ($\infty \mapsto \infty$)

Described by **roots** of **g**

$$\Big(f(x)/g(x), y \cdot h(x)/k(x) \Big)$$

Isogeny degree $\sim \#\,\mathrm{roots}(g)$

$\sim \deg(g)$

$$\{\ (x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + \overline{A} \cdot x^2 + x\ \} \cup \{\infty\}$$

Isogenies: Rational Maps between Elliptic Curves

$$\{\ (x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + A \cdot x^2 + x\ \} \cup \{\infty\}$$

Isogeny $(\infty \mapsto \infty)$

Described by **roots** of **g**

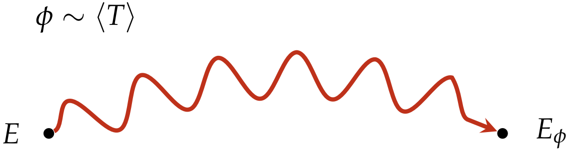
$$\left(f(x)/g(x), y \cdot h(x)/k(x) \right)$$

Cyclic isogeny
when $\ker(g) = \langle T \rangle$

Isogeny degree $\sim \#\,\mathrm{roots}(g)$
$\sim \deg(g)$

$$\{\ (x,y) \in \overline{\mathbb{F}}_p \times \overline{\mathbb{F}}_p \mid y^2 = x^3 + \overline{A} \cdot x^2 + x\ \} \cup \{\infty\}$$

Compression and Dual Isogenies



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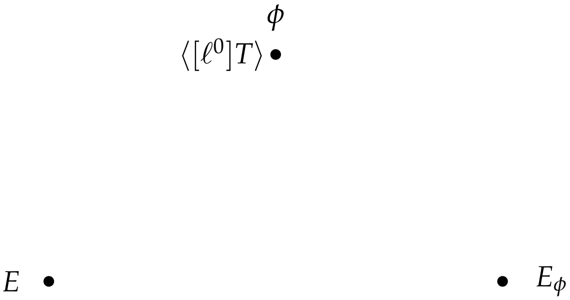

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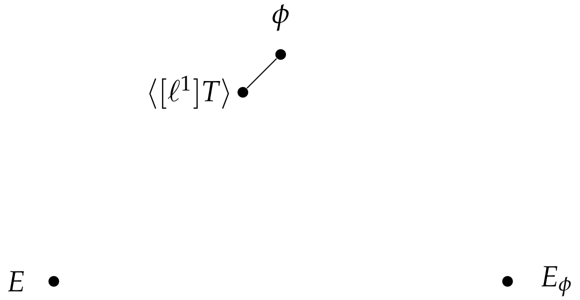

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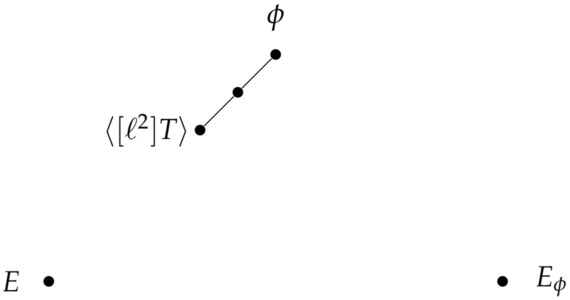

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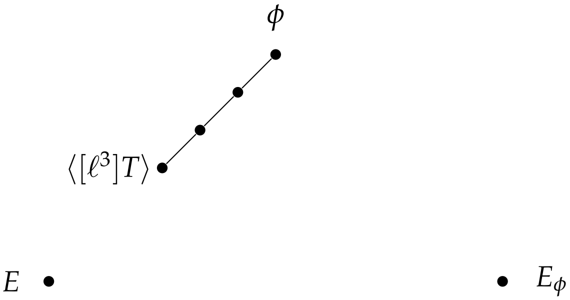

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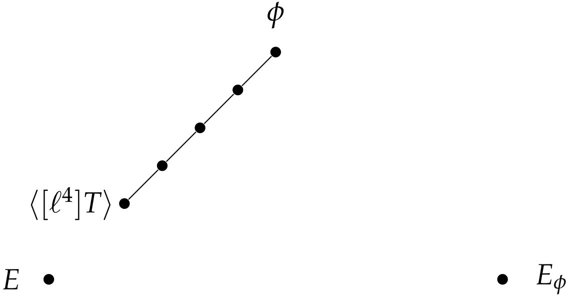

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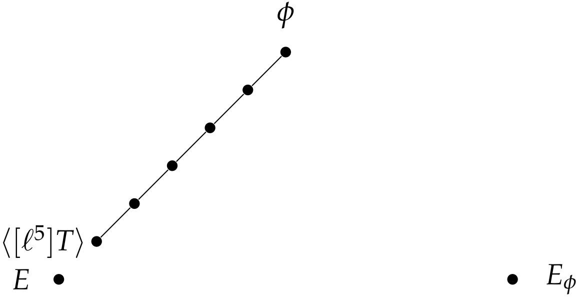

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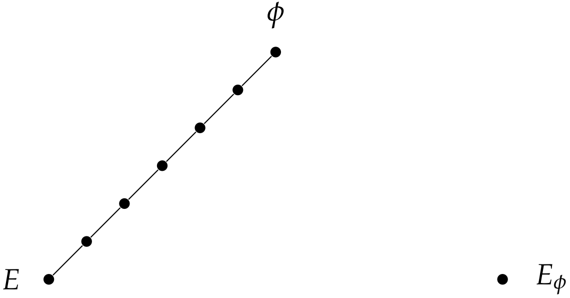

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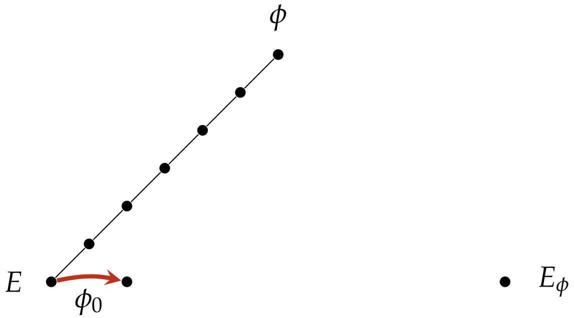

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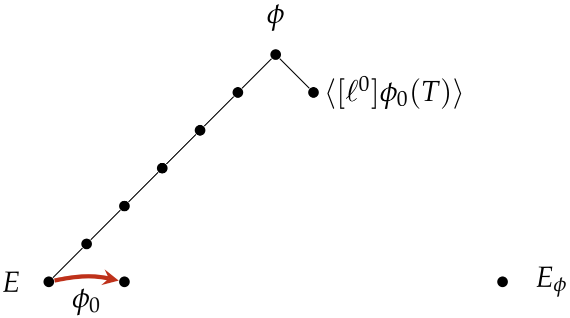

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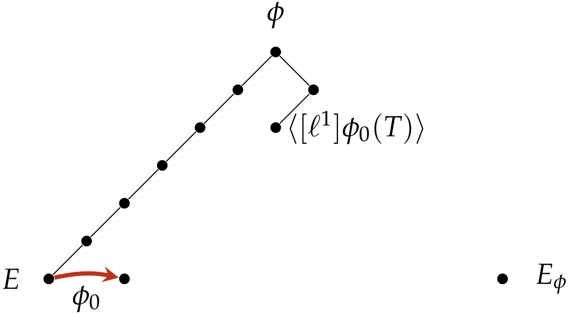

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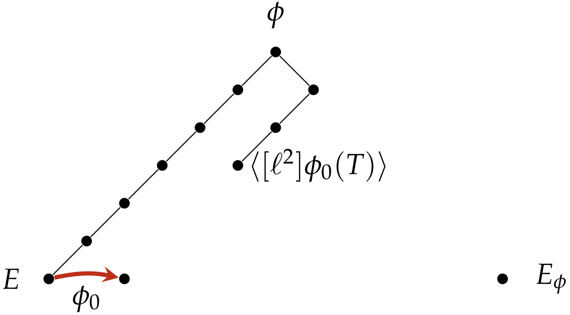

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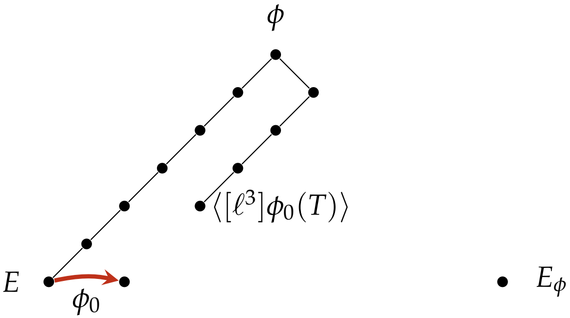

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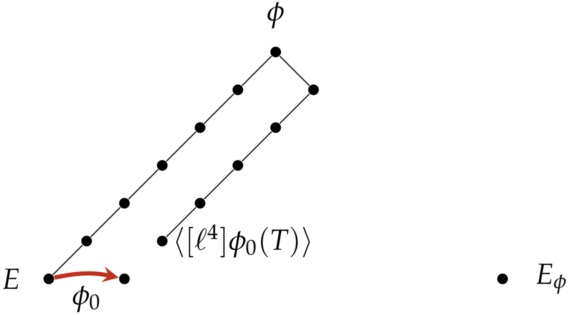

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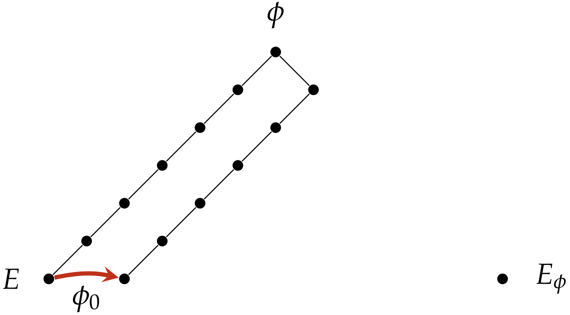

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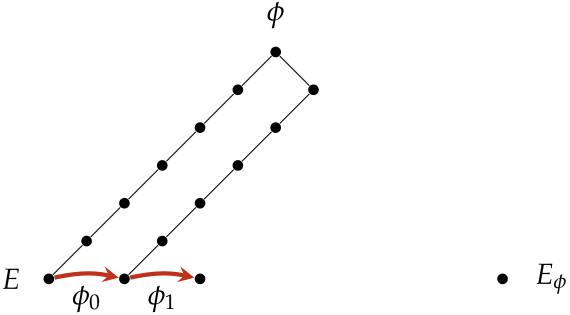

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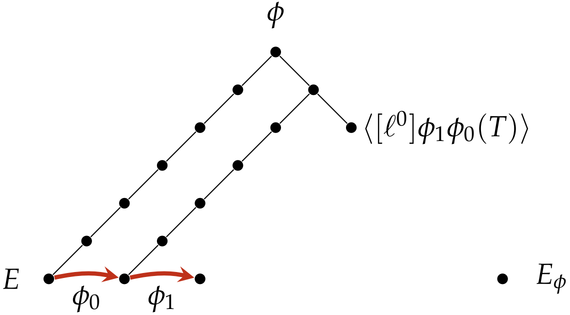

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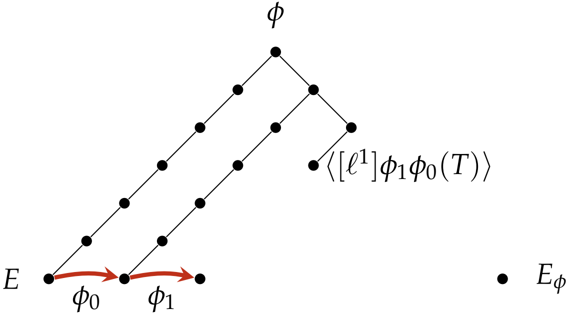

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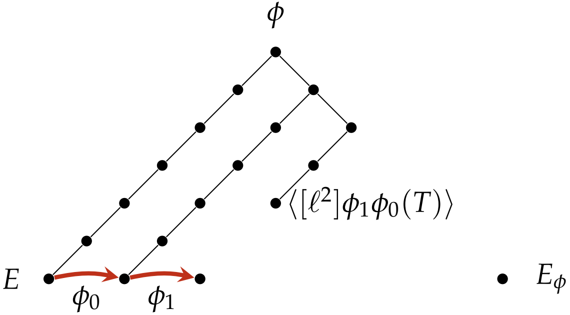

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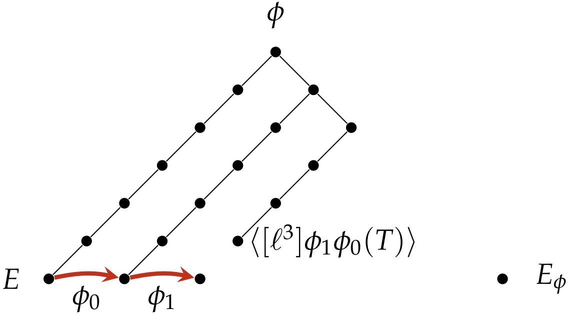

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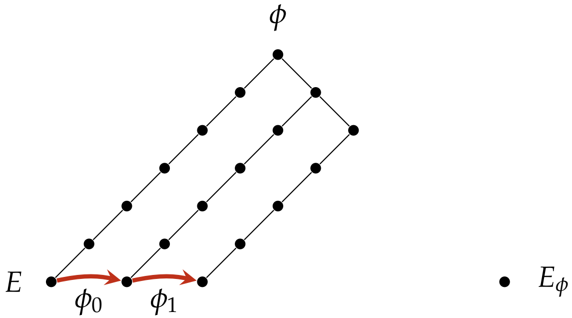

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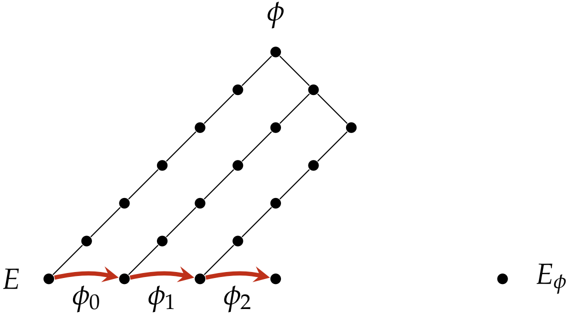

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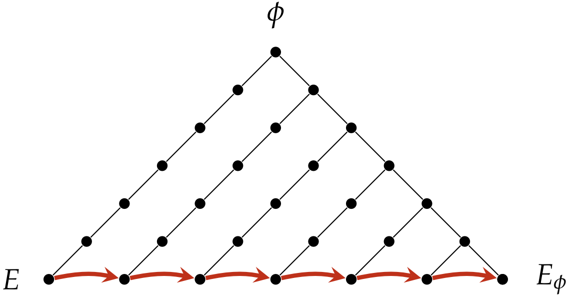

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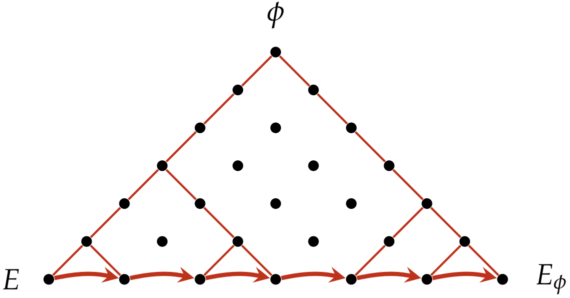

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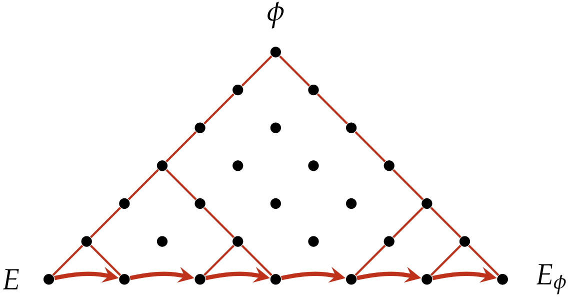
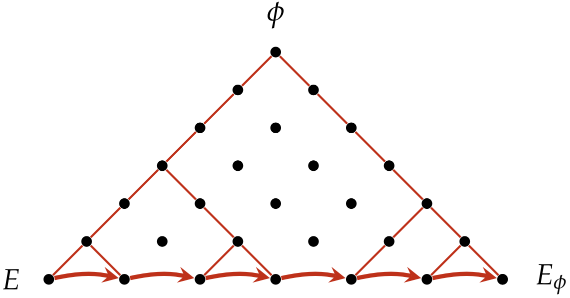


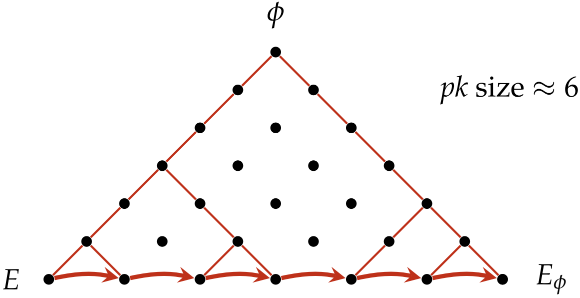
$$\begin{pmatrix} P \\ Q \end{pmatrix}$$

Compression and Dual Isogenies



$$\begin{pmatrix} P \\ Q \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix}$$

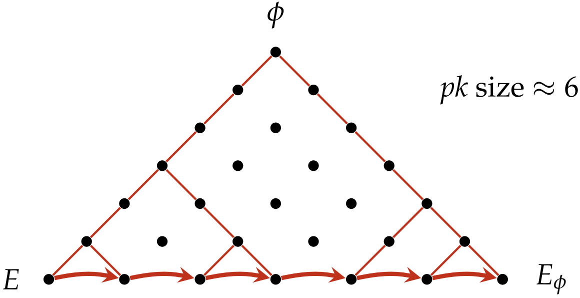
Compression and Dual Isogenies



pk size $\approx 6 \log p$

$$\begin{pmatrix} P \\ Q \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix}$$

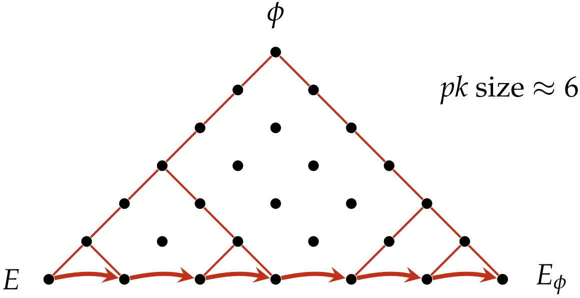
Compression and Dual Isogenies



pk size $\approx 6 \log p$

$$\begin{pmatrix} P \\ Q \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} \qquad \begin{pmatrix} R \\ S \end{pmatrix}$$

Compression and Dual Isogenies

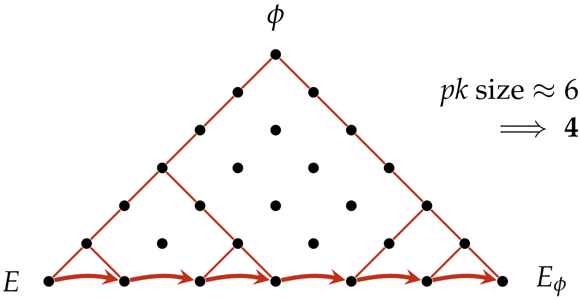


pk size $\approx 6 \log p$

$$\begin{pmatrix} P \\ Q \end{pmatrix}$$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

Compression and Dual Isogenies

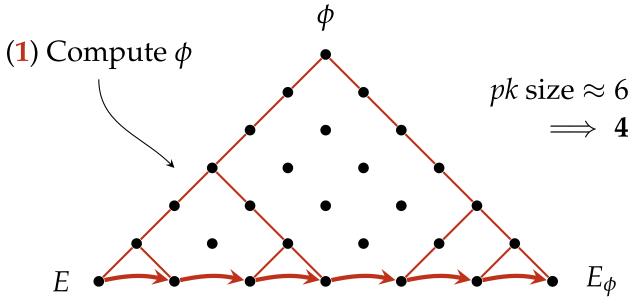


pk size $\approx 6 \log p$
$\implies$ $\mathbf{4} \log p$

$$\begin{pmatrix} P \\ Q \end{pmatrix}$$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

Compression and Dual Isogenies



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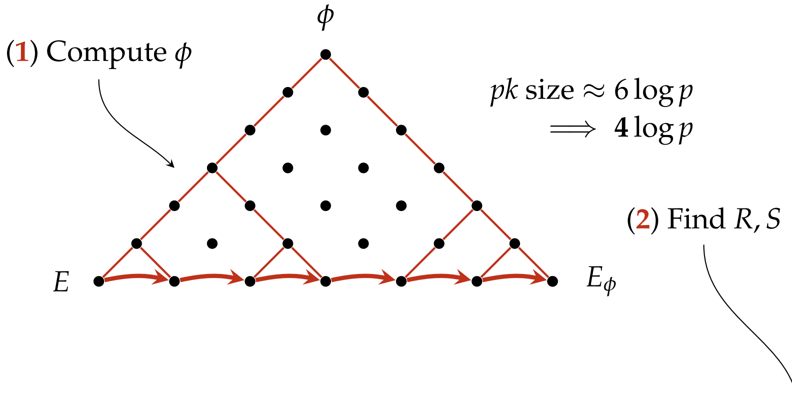
Compression and Dual Isogenies



pk size $\approx 6 \log p$

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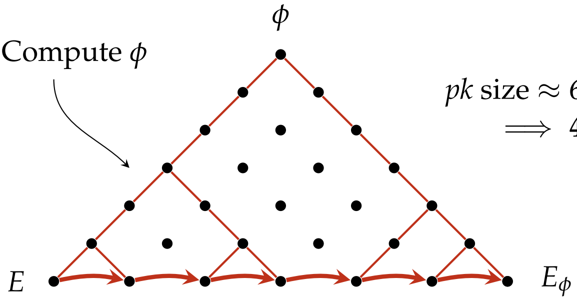
Compression and Dual Isogenies

(**1**) Compute ϕ



pk size $\approx 6 \log p$

$\implies$ **4** $\log p$

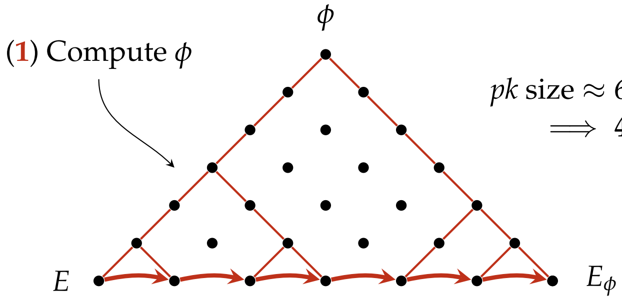
(**2**) Find R, S

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} P \\ Q \end{pmatrix}$$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

Compression and Dual Isogenies



pk size $\approx 6 \log p$

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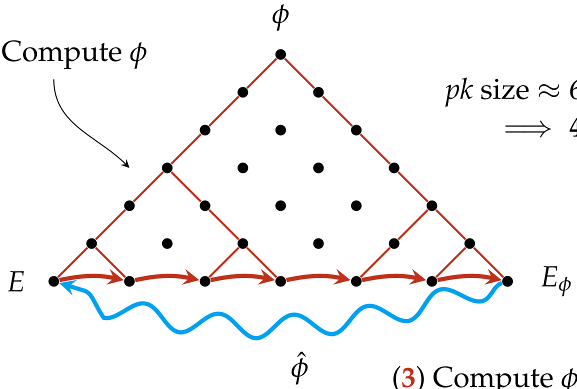
$$\begin{pmatrix} P \\ Q \end{pmatrix}$$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

Compression and Dual Isogenies

(**1**) Compute ϕ



pk size $\approx 6\log p$

$\implies \mathbf{4}\log p$

(**2**) Find R, S

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

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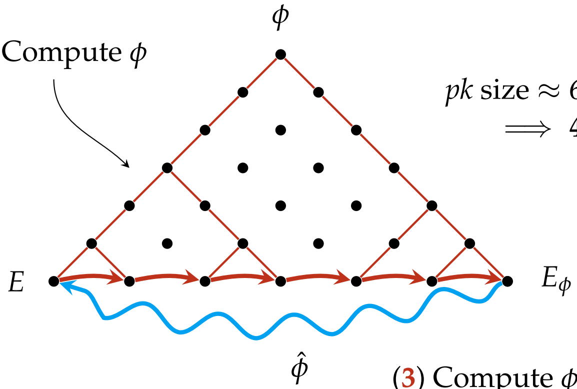
Compression and Dual Isogenies

(**1**) Compute ϕ



pk size $\approx 6 \log p$

$\implies$ $\mathbf{4} \log p$

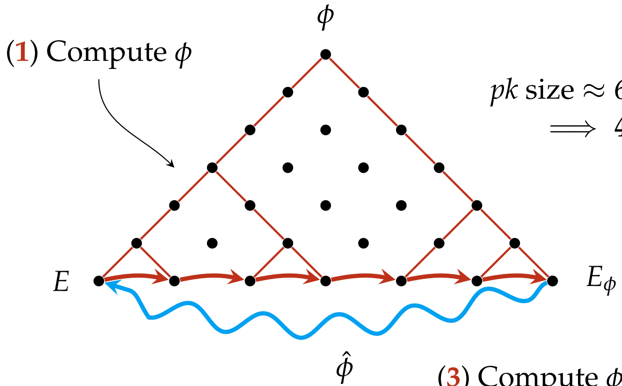
(**2**) Find R, S

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} \qquad \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

Compression and Dual Isogenies



pk size $\approx 6 \log p$

$\implies$ **4** $\log p$

(2) Find R, S

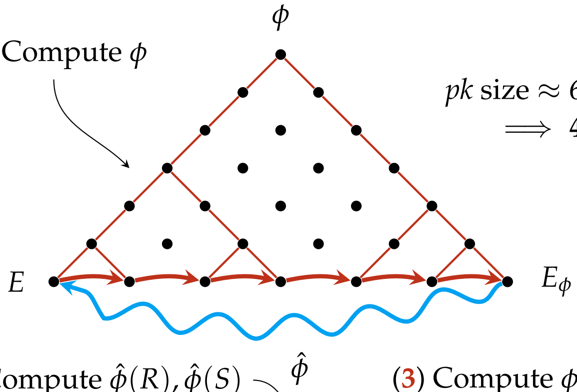
(3) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix} \qquad \begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(4) Find $\alpha, \beta, \gamma, \delta$

Compression and Dual Isogenies

(**1**) Compute ϕ



pk size $\approx 6\log p$
$\implies \mathbf{4}\log p$

(**2**) Find R, S

(**3**) Compute $\hat{\phi}(R), \hat{\phi}(S)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix}$$

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

Compression and Dual Isogenies

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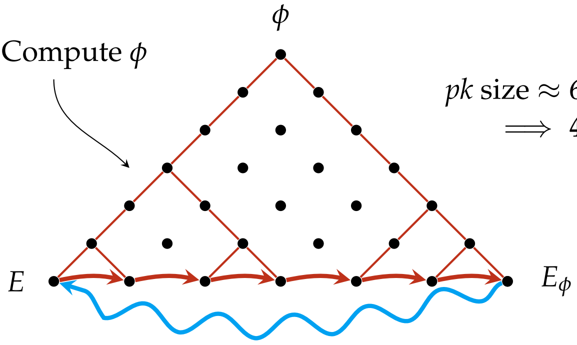

pk size $\approx 6 \log p$
$\implies$ **4** $\log p$

(**2**) Find R, S

$\hat{\phi}$

(**3**) Compute $\hat{\phi}(R), \hat{\phi}(S)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

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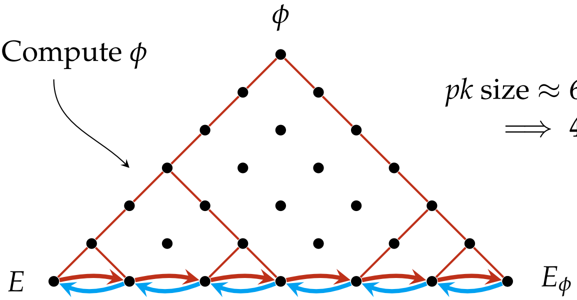

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(**2**) Find R, S

(**3**) Compute $\hat{\phi}(R), \hat{\phi}(S)$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

(**3**) Compute $\phi(P), \phi(Q)$

$$\begin{pmatrix} \phi(P) \\ \phi(Q) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}$$

(**4**) Find $\alpha, \beta, \gamma, \delta$

Speedups/slowdowns for (1) – (3)

	ℓ	p434	p503	p610	p751
SIKE-2	2	9 649	13 332	24 238	35 294
This	2	**7 921**	**11 039**	**20 269**	**30 922**
SIKE-2	3	7 062	9 859	16 830	28 258
This	3	**7 368**	**10 211**	**17 497**	**29 397**

Table 1: Efficiency for isogeny + basis gen. in 10^3 cycles on Skylake.

Supersingular Isogeny Key Encapsulation – `https://sike.org`.

(4) Find $\alpha, \beta, \gamma, \delta$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix}$$

(4) Find $\alpha, \beta, \gamma, \delta$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} = \qquad \begin{pmatrix} \hat{\phi}(R) \\ \hat{\phi}(S) \end{pmatrix}$$

(4) Find $\alpha, \beta, \gamma, \delta$

$$[a]P + [b]Q = \hat{\phi}(R)$$
$$[c]P + [d]Q = \hat{\phi}(S)$$

(4) Find $\alpha, \beta, \gamma, \delta$

$$[a]P + [b]Q = \hat{\phi}(R)$$
$$[c]P + [d]Q = \hat{\phi}(S)$$

$$\begin{aligned}
\tau(Q, \hat{\phi}(R)) &= \tau(P, Q)^{-a} \\
\tau(P, \hat{\phi}(R)) &= \tau(P, Q)^{b} \\
\tau(Q, \hat{\phi}(S)) &= \tau(P, Q)^{-c} \\
\tau(P, \hat{\phi}(S)) &= \tau(P, Q)^{d}
\end{aligned}$$

(4) Find $\alpha, \beta, \gamma, \delta$

$$[a]P + [b]Q = \hat{\phi}(R)$$
$$[c]P + [d]Q = \hat{\phi}(S)$$

$$\begin{aligned}
\tau(Q, \hat{\phi}(R)) &= \tau(P, Q)^{-a} \\
\tau(P, \hat{\phi}(R)) &= \tau(P, Q)^{b} \\
\tau(Q, \hat{\phi}(S)) &= \tau(P, Q)^{-c} \\
\tau(P, \hat{\phi}(S)) &= \tau(P, Q)^{d}
\end{aligned}$$

(reduced) **Tate** pairing

(4) Find $\alpha, \beta, \gamma, \delta$

$$[a]P + [b]Q = \hat{\phi}(R)$$
$$[c]P + [d]Q = \hat{\phi}(S)$$

$$\begin{aligned} \tau(Q, \hat{\phi}(R)) &= \tau(P, Q)^{-a} \\ \tau(P, \hat{\phi}(R)) &= \tau(P, Q)^{b} \\ \tau(Q, \hat{\phi}(S)) &= \tau(P, Q)^{-c} \\ \tau(P, \hat{\phi}(S)) &= \tau(P, Q)^{d} \end{aligned}$$

(reduced) **Tate** pairing

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

  $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$

end for

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

  $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$

end for

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

$g_{[2]P}$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
  end for

$g_{[2]P}$
$g_{[4]P}$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$

end for

$g_{[2]P}$

$g_{[4]P}$

$g_{[8]P}$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
  end for

$\longrightarrow$

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
  end for

Output: $\tau(P, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
  end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
  end for

Output: $\tau(P, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
  end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(a) Store table of $g_{[2^i]P}$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
end for

Output: $\tau(P, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
end for

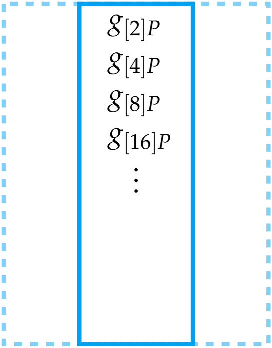
$g_{[2]P}$
$g_{[4]P}$
$g_{[8]P}$
$g_{[16]P}$
$\vdots$

(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(4) Quadruple Tate pairing

Output: $\tau(P, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(R))$
  end for

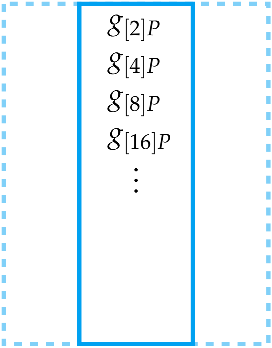
Output: $\tau(P, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]P}(\hat{\phi}(S))$
  end for



(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(4) Quadruple Tate pairing

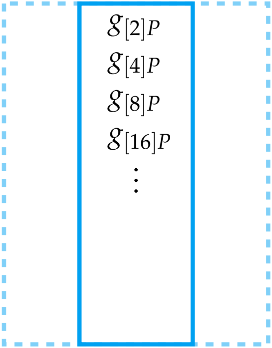
Output: $\tau(Q, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{[2^i]Q}(\hat{\phi}(R))$
end for



(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{[2^i]Q}(\hat{\phi}(R))$
  end for



(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(4) Quadruple Tate pairing

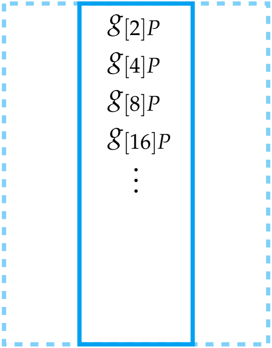
Output: $\tau(Q, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

$\quad f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$

end for



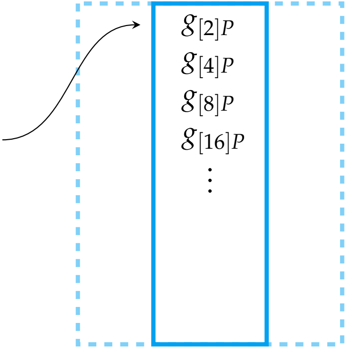
(a) Store table of $g_{[2^i]P}$

(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
  end for



(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

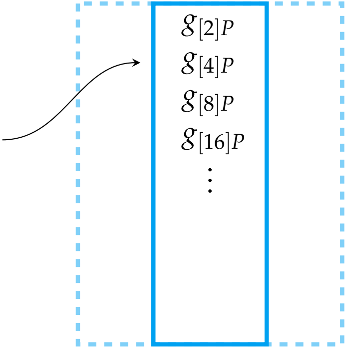
Output: $\tau(Q, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

  $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$

end for



(a) Store table of $g_{[2^i]P}$

(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

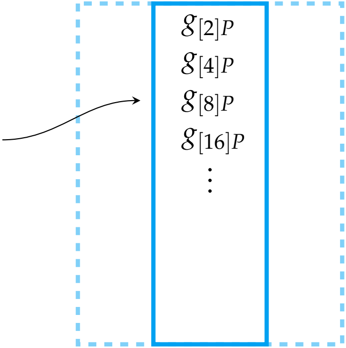
Output: $\tau(Q, \hat{\phi}(R))$

$f \leftarrow 1$

for $i = 1$ **to** e **do**

$f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$

end for



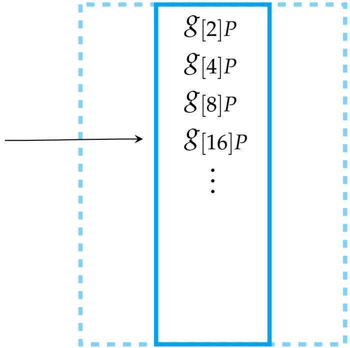
(a) Store table of $g_{[2^i]P}$

(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$

(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
  end for

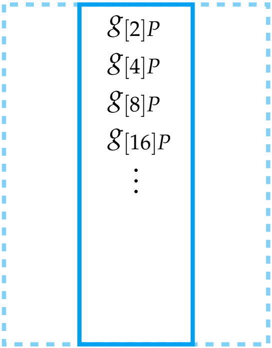


(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
  end for

Output: $\tau(Q, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(S))$
  end for

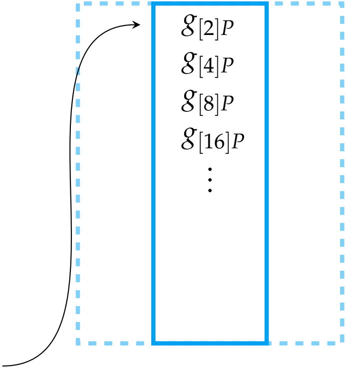


(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
  end for

Output: $\tau(Q, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(S))$
  end for

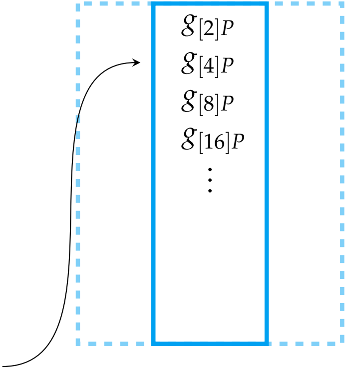


(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
end for

Output: $\tau(Q, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(S))$
end for

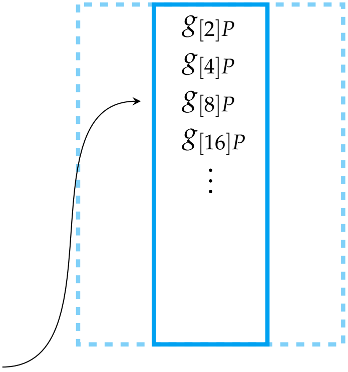


(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
end for

Output: $\tau(Q, \hat{\phi}(S))$
$f \leftarrow 1$
for $i = 1$ **to** e **do**
$f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(S))$
end for

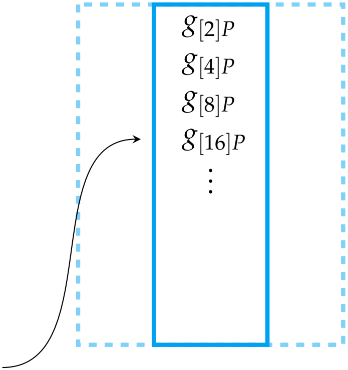


(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

(4) Quadruple Tate pairing

Output: $\tau(Q, \hat{\phi}(R))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(R))$
  end for

Output: $\tau(Q, \hat{\phi}(S))$
  $f \leftarrow 1$
  for $i = 1$ **to** e **do**
    $f \leftarrow f^2 \cdot g_{\psi([2^i]P)}(\hat{\phi}(S))$
  end for



(a) Store table of $g_{[2^i]P}$
(b) $P = (x, y)$ in $\mathbb{F}_p \times \mathbb{F}_p$
(c) $Q = \psi(P) = (-x, iy)$

Speedups for (4)

	ℓ	p434	p503	p610	p751
SIKE-2	2	5 821	8 033	13 458	21 908
This		**1 954**	**2 676**	**4 525**	**7 348**
SIKE-2	3	4 921	6 716	11 365	18 224
This		**1 821**	**2 486**	**4 214**	**6 727**

Table 2: Efficiency of pairing in 10^3 cycles on Skylake.

Supersingular Isogeny Key Encapsulation – `https://sike.org`.

Speedups for SIKE

	pk	KeyGen	Encaps	Decaps
SIKE-2	330 B	6 482	10 563	11 290
SIKE-2-comp	196 B	16 397	20 056	18 622
This	196 B	**10 849**	**16 600**	**15 682**

Table 3: Efficiency of KEM in 10^3 cycles on Skylake for `p434`.

Supersingular Isogeny Key Encapsulation – `https://sike.org`.

Speedups for SIKE

	pk	KeyGen	Encaps	Decaps
SIKE-2	—	—	—	—
SIKE-2-comp	−41%	153%	90%	65%
This	−41%	**67%**	**57%**	**39%**

Table 3: Efficiency of KEM in percentage on Skylake for `p434`.

Supersingular Isogeny Key Encapsulation – `https://sike.org`.

Thanks for your atthention!