

μ Kummer: efficient hyperelliptic signatures and key exchange on microcontrollers

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- ▶ Introduction
- ▶ High level signature and key exchange schemes
- ▶ Jacobian and Kummer arithmetic
- ▶ Implementation details
- ▶ Results and comparison

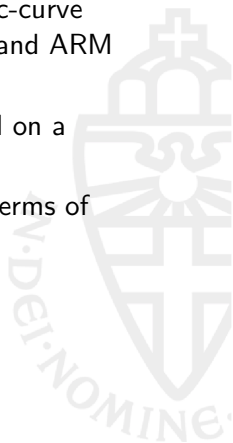


Summary of contributions

- 1 First software-only implementation of hyperelliptic-curve cryptography on microcontrollers (AVR ATmega and ARM Cortex M0)
- 2 First implementation of a signature scheme based on a Kummer surface
- 3 Significant improvement over state-of-the-art in terms of speed, size and stack usage

Software in the public domain. Available at

<http://www.cs.ru.nl/~jrenes/>



Curve-based cryptography

Genus	$g = 1$	$g = 2$
Curve	Elliptic curve E	Hyperelliptic curve \mathcal{E}
Cryptographic group	Points	Jacobian
Kummer	$E / \{\pm 1\}$	$\mathcal{K} := \mathcal{J} / \{\pm 1\}$



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- ▶ Operations

$$\text{DBL} : P \mapsto [2]P$$

$$\text{ADD} : P, Q \mapsto P + Q$$

- ▶ Two main use cases:

- ▶ Key exchange: relies on **scalar multiplication** $k, P \rightarrow [k]P$
- ▶ Signatures: relies on scalar multiplication and **addition**

- ▶ Operations on \mathcal{J} are hard to make fast and constant-time!

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- ▶ Corresponds to $(x, y) \mapsto x$
- ▶ **Not a group.** Use **x-only** operations

$$\text{xDBL} : x_P \mapsto x_{[2]P}$$

$$\text{xADD} : x_P, x_Q, x_{P \pm Q} \mapsto x_{P \mp Q}$$

- ▶ Scalar multiplication via the Montgomery ladder (e.g. Curve25519 [Ber06])
- ▶ Main use case: **key exchange**
- ▶ No signatures (e.g. Ed25519 [Ber+12])

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- ▶ Scalar multiplication via the Montgomery ladder
- ▶ Main use case: **key exchange**
- ▶ No signatures (need Jacobian)



(Hyper)elliptic curve crypto summarized

The situation in short:

- ▶ $E \leftrightarrow \mathcal{J}$
 - ▶ Key exchange ✓
 - ▶ Signatures ✓
- ▶ $E/\{\pm 1\} \leftrightarrow \mathcal{K}$
 - ▶ Key exchange ✓
 - ▶ Signatures ✗

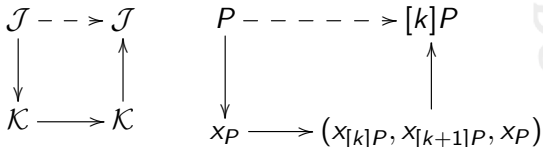


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- ▶ $E/\{\pm 1\} \leftrightarrow \mathcal{K}$
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New result [CCS16]; use \mathcal{K} to do fast signatures on \mathcal{J} :



PpR: "Project-pseudomultiply-Recover"

Implementation results

- ▶ On larger platforms speed records are challenged by Kummer surface implementations [CL15; Ber+14]
- ▶ Speed records for 128-bit secure key exchange and signatures on **microcontrollers** held by elliptic-curve-based schemes

Two interesting questions:

- ▶ Q: How well do Kummer-based key exchange schemes perform on microcontrollers?
 - A: *Probably well, but never implemented*
- ▶ Q: How do Kummer-based signatures schemes perform?
 - A: *Not clear*

The signature scheme

- ▶ Public generator $P \in \mathcal{J}$, 512-bit hash function H , 256-bit secret key d , message M
- ▶ Three main functions
 - ▶ keygen:
 - ① $(d' || d'') \leftarrow H(d)$
 - ② $Q \leftarrow [16d']P$
 - ▶ sign:
 - ① $(d' || d'') \leftarrow H(d)$
 - ② $r \leftarrow H(d'' || M)$
 - ③ $R \leftarrow [r]P$
 - ④ $h \leftarrow H(R || Q || M)$
 - ⑤ $s \leftarrow r - 16h_{128}d' \pmod{\#\mathcal{J}/16}$
 - ⑥ $\sigma \leftarrow (h_{128} || s)$
 - ▶ verify:
 - ① $T \leftarrow [s]P + [h_{128}]Q$
 - ② $g \leftarrow H(T || Q || M)$
 - ③ $g_{128} \stackrel{?}{=} h_{128}$



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 - ③ $R \leftarrow [r]P$
 - ④ $h \leftarrow H(R || Q || M)$
 - ⑤ $s \leftarrow r - 16h_{128}d' \pmod{\#\mathcal{J}/16}$
 - ⑥ $\sigma \leftarrow (h_{128} || s)$ (!) Compressed to 384 bits by sending h_{128}
 - ▶ verify:
 - ① $T \leftarrow [s]P + [h_{128}]Q$ (!) Half-size scalar multiplication
 - ② $g \leftarrow H(T || Q || M)$
 - ③ $g_{128} \stackrel{?}{=} h_{128}$

The signature scheme

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- ▶ Three main functions
 - ▶ keygen:
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 - ② $Q \leftarrow [16d']P$ (!) Compression of Q
 - ▶ sign:
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 - ③ $R \leftarrow [r]P$ (!) Compression of R
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The key exchange scheme

- ▶ Public generator $P \in \mathcal{K}$, 256-bit secret key d
- ▶ One main function
 - ▶ dh_exchange:
 - 1 $Q \leftarrow [d]P$



The key exchange scheme

- ▶ Public generator $P \in \mathcal{K}$, 256-bit secret key d
- ▶ One main function
 - ▶ dh_exchange:
 - ① $Q \leftarrow [d]P$ (!) Only on \mathcal{K}



The key exchange scheme

- ▶ Public generator $P \in \mathcal{K}$, 256-bit secret key d
- ▶ One main function
 - ▶ dh_exchange:
 - ① $Q \leftarrow [d]P$ (!) Both keygen and exchange



Building blocks: Jacobian & Kummer

- ▶ Finite field \mathbb{F}_q with $q = 2^{127} - 1$
- ▶ The Gaudry-Schost curve \mathcal{C} is a genus 2 hyperelliptic curve

$$\mathcal{C} : Y^2 = X(X - 1)(X - \lambda)(X - \mu)(X - \nu),$$

for constants $\lambda, \mu, \nu \in \mathbb{F}_q$

- ▶ Jacobian $\mathcal{J}_{\mathcal{C}}(\mathbb{F}_q)$
- ▶ Kummer surface $\mathcal{K}_{\mathcal{C}}(\mathbb{F}_q) := \mathcal{J}_{\mathcal{C}}(\mathbb{F}_q) / \{\pm 1\}$

Function	Domain & Range	M	S	m_c	a	s	l
ADD	$\mathcal{J}_{\mathcal{C}} \rightarrow \mathcal{J}_{\mathcal{C}}$	28	2	0	11	24	0
Project	$\mathcal{J}_{\mathcal{C}} \rightarrow \mathcal{K}_{\mathcal{C}}$	8	1	4	7	8	0
xDBLADD	$\mathbb{Z} \times \mathcal{K}_{\mathcal{C}} \rightarrow \mathcal{K}_{\mathcal{C}}^2$	7	12	12	16	16	0
Recover	$\mathcal{J}_{\mathcal{C}} \times \mathcal{K}_{\mathcal{C}}^3 \rightarrow \mathcal{J}_{\mathcal{C}}$	77	8	0	19	10	1

AVR ATmega

- ▶ Family of 8-bit microcontrollers
- ▶ Represent elements of $\mathbb{F}_{2^{127}-1}$ with 16 8-bit words (1 bit left)
- ▶ 128×128-bit multiplication (`bigint_mul`) and squaring (`bigint_sqr`) from [HS15]
 - ▶ 2-level Karatsuba multiplication and 1-level Karatsuba squaring
- ▶ Reduction (`bigint_red`) based on $2^{128} \equiv 2 \pmod{2^{127}-1}$
- ▶ Combined into field multiplication (`gfe_mul`) and squaring (`gfe_sqr`)
- ▶ Fast 16×128-bit multiplication by constant (`gfe_mulconst`)
- ▶ Inversion (`gfe_invert`) based on $g^{-1} = g^{2^{127}-3}$

ARM Cortex M0

- ▶ 32-bit microcontroller
- ▶ Represent elements of $\mathbb{F}_{2^{127}-1}$ with 4 32-bit words (1 bit left)
- ▶ 128×128-bit multiplication (`bigint_mul`) and squaring (`bigint_sqr`) from [Dül+15]
 - ▶ 2-level Karatsuba multiplication and 2-level Karatsuba squaring
- ▶ Reduction (`bigint_red`) based on $2^{128} \equiv 2 \pmod{2^{127}-1}$
- ▶ Combined into field multiplication (`gfe_mul`) and squaring (`gfe_sqr`)
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AVR ATmega (scalarmult)

	Imp.	Object	Cycles	Code size	Stack
DH	[LWG14]	256-bit curve	$\approx 21\,078\,200$	14 700 bytes	556 bytes
S,DH	[WUW13]	NIST P-256	$\approx 34\,930\,000$	16 112 bytes	590 bytes
DH	[HS13]	Curve25519	22 791 579	n/a	677 bytes
DH	[Dül+15]	Curve25519	13 900 397	17 710 bytes	494 bytes
DH	This work	\mathcal{K}_c	9 513 536	$\approx 9\,490$ bytes	99 bytes
S	This work	\mathcal{J}_c	9 968 127	$\approx 16\,516$ bytes	735 bytes

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Key exchange: Reducing number of clock cycles by 32%, almost halving code size and reducing stack usage by about 80%

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Signatures: Reducing number of clock cycles by 71%, increasing stack usage by 25%

AVR ATmega (full signatures)

Imp.	Object	Function	Cycles	Stack
[NLD15]	Ed25519	sig. gen.	19 047 706	1 473 bytes
[NLD15]	Ed25519	sig. ver.	30 776 942	1 226 bytes
This work	\mathcal{I}_c	sign	10 404 033	926 bytes
This work	\mathcal{I}_c	verify	16 240 510	992 bytes

Almost half the number of cycles, decrease stack usage (code size not reported)

ARM Cortex M0 (scalarmult)

	Imp.	Object	Clock cycles	Code size	Stack
S,DH	[WUW13]	NIST P-256	$\approx 10\,730\,000$	7 168 bytes	540 bytes
DH	[Dül+15]	Curve25519	3 589 850	7 900 bytes	548 bytes
DH	This work	\mathcal{K}_c	2 633 662	$\approx 4\,328$ bytes	248 bytes
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DH	This work	\mathcal{K}_c	2 633 662	$\approx 4\,328$ bytes	248 bytes
S	This work	\mathcal{J}_c	2 709 401	$\approx 9\,874$ bytes	968 bytes

Key exchange: Reducing number of clock cycles by 27%, halving code size and stack usage

ARM Cortex M0

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S, DH	[WUW13]	NIST P-256	$\approx 10\,730\,000$	7 168 bytes	540 bytes
DH	[Dül+15]	Curve25519	3 589 850	7 900 bytes	548 bytes
DH	This work	\mathcal{K}_c	2 633 662	$\approx 4\,328$ bytes	248 bytes
S	This work	\mathcal{J}_c	2 709 401	$\approx 9\,874$ bytes	968 bytes

Signatures: Reducing number of clock cycles by 75%, increase in code size and stack usage

Thanks for your attention!



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